MLE & Method of Moments (MOM) – Reading & Comments

Read Chapter 9 of Kay on the Method of Moments. Also read Kay Chapter 7 and Moon Sections 12.1–12.4 on Maximum Likelihood Estimation (MLE). Maximum Likelihood Estimation (MLE) is perhaps the most commonly used technique for parameter estimation. Like Least-Squares Estimation (LSE), MLE is a key concept and technique that you should study well and assume that you will encountered repeatedly throughout your engineering career.\footnote{Recall that Least-Squares Estimation can be interpreted as a special case of MLE under a gaussian, linear model assumption.} Compared to the potential difficulties associated with obtaining a UMVUE via the Rao-Blackwell theory (to be discussed in ECE275B next quarter), the MLE methodology is the height of \textit{conceptual} simplicity.\footnote{Of course, \textit{Numerical} issues can be another story. Ergo the keen interest in the Expectation Maximization (EM) algorithm in recent years. The EM algorithm will be also discussed next quarter.}

The MLE approach is based on a heuristically reasonable inductive principle, but otherwise is not designed at the outset to result in estimators with any optimality properties. Thus the existence of such properties (if they do occur) can be viewed as a “happy accident” for the purposes of our class.\footnote{Advanced statistics courses analyze under just what conditions a regular statistical family will have such desirable properties.} Be sure to understand the \textit{asymptotic MLE properties} of asymptotic unbiasedness, asymptotic efficiency, consistency, and asymptotic normality. Note that asymptotic efficiency and asymptotic normality together allow one to readily compute asymptotically valid (so-called) \textit{confidence intervals} that overlap the unknown parameter vector with a specified degree of probability, thereby providing a rational way to assign “error bars” to our point estimates.

Because of time constraints, we do not have time in lecture to provide even heuristically plausible proofs for many of the MLE properties. A course in Mathematical Statistics is highly recommended for those of you who desire a rigorous derivation and justification of these properties.

Comments on the Final Exam

The final exam will cover all of the material presented in class and lecture through, and including, this homework 7, the midterm and sample (undergraduate and graduate) midterms, and the lecture supplements. The final exam is closed notes and closed book. You are to bring your own paper and pens. No electronic devices (calculators, cell phones, etc.) will be allowed.
Problems

1. Let the real scalar random variable $y$ be uniformly distributed over the closed interval $[0, \theta]$, for unknown $\theta$, $0 < \theta < \infty$. (Note that this is the same model used in Homework Problem 6.11.) Given iid measurement samples, $y_1, \cdots, y_m$ drawn from this distribution, use the method of moments to derive an estimate of $\theta$. Is this estimate unbiased? Is it consistent? (Prove your answers.)

2. Let the real scalar random variable $y$ be uniformly distributed over the closed interval $[-\theta, \theta]$ (note the change from the previous problem), for unknown $\theta$, $0 < \theta < \infty$. Given iid measurement samples, $y_1, \cdots, y_m$ drawn from this distribution, use the method of moments to derive an estimate of $\theta$. Is this estimate unbiased? Is it consistent? (Prove your answers.)

3. Kay 7.3

4. Kay 7.8

5. Kay 7.9

6. Kay 7.12

7. Prove the Invariance Principle of Maximum Likelihood Estimation (Kay Theorem 7.4) assuming that $g(\cdot)$ is many–to–one.

8. Kay 7.17