\[ H_2(z) = 1 - z^{-1} + 2z^{-2} - 0.5z^{-3} + z^{-4} - 2z^{-5} + z^{-7} + z^{-10} \]
\[ H_3(z) = 1 - jz^{-1} - 2z^{-2} + 0.5jz^{-3} + z^{-4} - 2jz^{-5} + jz^{-7} - z^{-10} \]

4.19.

a) Using the numerical values given we get
\[ F_0(z) = 2 + z^{-1} + z^{-2} + 3z^{-3} + z^{-5} - z^{-7} \]
\[ F_1(z) = 2W^2 + Wz^{-1} + z^{-2} + 3W^2z^{-3} + z^{-5} - Wz^{-7} \]
\[ F_2(z) = 2W + W^2z^{-1} + z^{-2} + 3Wz^{-3} + z^{-5} - W^2z^{-7} \]
where \( W = e^{-2\pi j/3} \).

b) Responses of filters \( F_0(z) \) and \( F_2(z) \) are as shown.

Fig. S4-19.

4.20. From Fig. 4.3-12,
\[ F_k(z) = \sum_{t=0}^{M-1} z^{-M+1+t(W^k)^t} \]
Letting \( M - 1 - i = p \), we get,
\[ F_k(z) = W^{-k}\sum_{p=0}^{M-1} (zW^k)^{-p} = W^{-k}H_0(zW^k). \]
4.21.

\[ E_o(z) = 1 + 4z^{-1} + z^{-2}, \quad E_i(z) = 2 + 2z^{-1} \]

Fig. S4-21.

4.22.

a) For \( M = 3 \), we get \( m_0 = 1 \). For \( M = 4 \), \( m_0 = 3 \).

b) By direct substitution, it can be verified that \( m_0 = N \mod M \) holds true.

4.23.

a) Using Kaiser’s order estimation formula for equiripple filters (3.2.32),

\[
N_p = \frac{-10\log(\delta_1\delta_2/2) - 13}{14.6M_1(\frac{\pi}{2\pi})} \quad (S4.23a)
\]

\[
N_i = \frac{-10\log(\delta_1\delta_2/2) - 13}{14.6(\frac{1}{M_1} - \frac{\pi}{2\pi})} \quad (S4.23b)
\]

From the equations, it is clear that as \( M_1 \) increases, \( N_p \) decreases but \( N_i \) will increase because \( M_1 \) occurs as a reciprocal in its denominator.

b) For the given numerical values, \( M_1 = 2, 3, 4, 5 \) are the permissible values. From the graph, we see that \( M_1 = 3 \) is the optimal value.

\[ \text{Multiplier Count} \]

\[ 114 \times \quad 110 \times \quad 115 \times \]

\[ 2 \quad 3 \quad 4 \quad 5 \]

Fig. S4-23.