PROBLEMS

4.1. Consider the structures shown in Fig. P4-1, with input transforms and filter responses as indicated.

Figure P4-1

Sketch the quantities $Y_0(e^{j\omega})$ and $Y_1(e^{j\omega})$.

4.2. For the system in Fig. P4-2, find an expression for $y(n)$ in terms of $z(n)$.

Simplify the expression as best as you can.

4.3. Consider a sequence $z(n)$ with $X(e^{j\omega})$ as shown in Fig. P4-3.

Let $y(n) = x(2n)$. Show how we can recover $z(n)$ from $y(n)$ using filters and multirate building blocks.

4.4. Show that the decimator and expander are linear time varying systems.

4.5. Show that the two systems shown in Fig. P4-5(a) (where $k$ is some integer) are equivalent (that is, $y_0(n) = y_1(n)$) when $h_k(n) = h_0(n) \cos(2\pi kn/L)$.
This is a structure where filtering followed by cosine modulation has the same effect as filtering with the cosine modulated impulse response. (This is not true in all situations; see next problem). Now consider the example where \( L = 5 \), and \( k = 1 \). Let \( X(e^{j\omega}) \) and \( H_0(e^{j\omega}) \) be as sketched in Fig. P4-5(b).

![Figure P4-5(b)](image)

Figure P4-5(b)

Give sketches of \( Y(e^{j\omega}), Y_0(e^{j\omega}) \) and \( U(e^{j\omega}) \).

4.6. Show that the two systems shown in Fig. P4-6 are not equivalent, that is, \( y_0(n) \) and \( y_1(n) \) are not necessarily the same, even if \( h_k(n) = h_0(n) \cos(2\pi kn/L) \).

Figure P4-6

4.7. Consider the two sets of \( M \) numbers given by \( W^k, 0 \leq k \leq M - 1 \) and \( W^{kL}, 0 \leq k \leq M - 1 \) where \( W = e^{-j2\pi/M} \). Show that these sets are identical if and only if \( L \) and \( M \) are relatively prime.

4.8. For the two systems in Fig. 4.2-2 we can write down \( y_1(n) \) and \( y_2(n) \) in terms of \( z(n), M \) and \( L \). For example

\[
y_1(n) = \begin{cases} z\left(\frac{Mn}{L}\right), & n = \text{mul. of } L \\ 0, & \text{otherwise.} \end{cases}
\]

a) Similarly write an expression for \( y_2(n) \).

b) Verify that these two expressions yield the same result (i.e., \( y_1(n) = y_2(n) \) for any sequence \( z(n) \)), if, and only if, \( L \) and \( M \) are relatively prime.

4.9. The jumping painter problem. Consider a circular arrangement of objects as shown in Fig. P4-9.