4.18. Consider the uniform DFT analysis bank [Fig. 4.3-5(a)] with $M = 4$. Assume $E_0(z) = 1 + z^{-1}, E_1(z) = 1 + 2z^{-1}, E_2(z) = 2 + z^{-2}$ and $E_3(z) = 0.5 + z^{-1}$. Find explicit expressions for $H_k(z), 0 \leq k \leq 3$, working out the numerical values of these filter coefficients.

4.19. Consider the structure shown in Fig. P4-19(a), where $W$ is the $3 \times 3$ DFT matrix.

![Structure Diagram](image)

This is a three channel synthesis bank with three filters $F_0(z), F_1(z)$ and $F_2(z)$. (For example $F_0(z) = Y(z)/Y_0(z)$ with $y_1(n)$ and $y_2(n)$ set to zero.)

a) Assuming $R_0(z) = 1 + z^{-1}, R_1(z) = 1 - z^{-2}$ and $R_2(z) = 2 + 3z^{-1}$, find expressions for the three synthesis filters $F_0(z), F_1(z), F_2(z)$.

b) Let the magnitude response of $F_1(z)$ be as shown in Fig. 4.19(b).

![Magnitude Response Diagram](image)

Plot the responses $|F_0(e^{i\omega})|$ and $|F_2(e^{i\omega})|$.

4.20. For the structure of Fig. 4.3-12, prove that the synthesis filters are indeed given by (4.3.15).

4.21. Let $H_0(z) = 1 + 2z^{-1} + 4z^{-2} + 2z^{-3} + z^{-4}$ and let $H_1(z) = H_0(-z)$. Draw an implementation for the pair $[H_0(z), H_1(z)]$ in the form of a uniform DFT analysis bank, explicitly showing the polyphase components, the $2 \times 2$ IDFT box, and other relevant details.

4.22. Let $H(z) = \sum_{n=0}^{N} h(n)z^{-n}$ with $h(n) = h(N-n)$. Consider the polyphase decomposition (4.3.7). The symmetry of $h(n)$ reflects into the coefficients of $E_k(z)$ in some way. To be more specific, we can make the following statement: there exists an integer $m_0$ (with $0 \leq m_0 \leq M - 1$) such that $\epsilon_k(n)$ is the image of $\epsilon_{m_0-k}(n)$ for $0 \leq k \leq m_0$, and $\epsilon_k(n)$ is the image of $\epsilon_{M+m_0-k}(n)$ for $m_0 + 1 \leq k \leq M - 1$.

a) Take an example of 7th order $H(z)$, and verify the above statement for $M = 3$. What is $m_0$? Repeat for $M = 4$.

b) *Prove* the above statement. How is $m_0$ related to $N$ and $M$?
b) Suppose the specifications for $H(z)$ are $\omega_p = 0.85\pi$, $\omega_s = 0.9\pi$, $\delta_1 = 0.01$, $\delta_2 = 0.001$. What is the total number of multipliers required if we design linear-phase equiripple $H(z)$ directly?

c) Suppose we meet the specifications of part (b) by proceeding as in part (a) where $H_1(z)$ is designed using the IFIR approach (stretching factor 2). What are the specifications for $H_1(z)$? What is the required number of multipliers for implementing the wideband filter $H(z)$? (Take the model filter and image suppressor to be equiripple.)

4.26. Suppose we wish to design a 25-fold lowpass linear-phase interpolation filter (i.e., $L = 25$). Let the input signal $x(n)$ be bandlimited to $|\omega| < 0.95\pi$. Assume that the ripple specifications are $\delta_1 = 0.01$, $\delta_2 = 0.002$. (a) Find reasonable band edges $\omega_p$ and $\omega_s$ for $H(z)$. (b) What is the filter order if a direct design is used? (c) Suppose the filter is designed using a two stage approach. What are the orders of $G(z)$ and $I(z)$? (d) What are the total number of multiplications and additions in the direct design and how do these compare with the two-stage design? (e) Assuming an input sampling rate of 8 KHz, what is the number of multiplications and additions per second in the two-stage design?

4.27. For a uniform DFT analysis bank, we know that the filters are related by $H_k(z) = H_0(zW^k)$, $0 \leq k \leq M - 1$, with $W = e^{-j2\pi/M}$. Let $M = 5$ and define two new transfer functions $G_1(z) = H_1(z) + H_4(z)$ and $G_2(z) = H_2(z) + H_3(z)$. Let $h_0(n)$ denote the impulse response of $H_0(z)$, assumed to be real for all $n$.

a) Are $h_k(n)$, $1 \leq k \leq 4$ real for all $n$?

b) Express the impulse responses $g_1(n)$ and $g_2(n)$ of $G_1(z)$ and $G_2(z)$ in terms of $h_0(n)$. Are $g_1(n)$ and $g_2(n)$ real for all $n$?

c) Let $|H_0(e^{j\omega})|$ be as shown in Fig. P4-27.

Plot the responses $|G_1(e^{j\omega})|$ and $|G_2(e^{j\omega})|$, for $0 \leq \omega \leq 2\pi$. Does $|G_2(e^{j\omega})|$ necessarily look ‘good’ in its passband?

4.28. Consider the analysis/synthesis system in Fig. P4-28.

a) Let the analysis filters be $H_0(z) = 1 + 3z^{-1} + 0.5z^{-2} + z^{-3}$ and $H_1(z) = H_0(-z)$. Find causal stable IIR filters $F_0(z)$ and $F_1(z)$ such that $\tilde{x}(n)$
agrees with $x(n)$ except for a possible delay and (nonzero) scale factor.

b) Let $H_0(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$, and $H_1(z) = H_0(-z)$. Find causal FIR filters $F_0(z)$ and $F_1(z)$ such that $\hat{x}(n)$ agrees with $x(n)$ except for a possible delay and (nonzero) scale factor.

Figure P4-28

(Hint. This is perhaps tricky, but not tedious or difficult. It helps to use polyphase decomposition. Review of complementary filters might help.)

4.29. Let $H_0(z) = (1 + z^{-1})/2$. Find real-coefficient causal FIR $H_1(z)$ such that the pair $H_0(z), H_1(z)$ is power complementary. Are these filters also allpass complementary? Euclidean complementary? Doubly complementary?

4.30. A trick for the design of zero-phase FIR equiripple half-band filters. Suppose $G(z) = \sum_{n=0}^{N} g(n)z^{-n}$ is a Type 2 linear phase filter (Sec. 2.4.2). This means that $N$ is odd and $g(n)$ is real, with $g(n) = g(N - n)$. This also means that there is a zero at $\omega = \pi$. We know we can write $G(e^{j\omega}) = e^{-j\omega N/2}G_R(\omega)$ where $G_R(\omega)$ real. Suppose we have designed $G(z)$ such that the response $G_R(\omega)$ is as shown in Fig. P4-30.

Figure P4-30

This design can be done by defining the passband to be $0 \leq \omega \leq \theta_p$ and transition band to be $\theta_p \leq \omega \leq \pi$. There is no stopband. Such filters with one equiripple passband and no stopband can indeed be designed using the McClellan-Parks algorithm (Section 3.2.4). Now define the transfer function $F(z) = [z^{-N} + G(z^2)]/2$. This is a Type 1 linear phase filter.

a) Show that $F(e^{j\omega}) = e^{-j\omega N}F_R(\omega)$, where $F_R(\omega)$ is real. Express $F_R(\omega)$ in terms of $G_R(\omega)$.

b) Plot the amplitude response $F_R(\omega)$ in $0 \leq \omega \leq \pi$. Verify that it resembles Fig. 4.6-4. What are the values of $\delta, \omega_p$, and $\omega_s$ in terms of $\epsilon$ and $\theta_p$?

c) Let $f(n)$ and $g(n)$ denote the impulse responses of $F(z)$ and $G(z)$. Show