L should be the smallest integer such that L is a multiple of q. Therefore,

\[ L = q = \frac{N}{p} = \frac{N}{\gcd(M, N)} = \frac{\text{lcm}(M, N)}{M}. \]

Obviously, when M and N are relatively prime, \( L = N \) is the largest possible value.

4.11. The plots are

![Fig. S4-11.](image)

4.12.

a) \( h(n) = (1/2)^n \) for \( 0 \leq n \leq 9 \). So

\[ E_0(z) = h(0) + h(2)z^{-1} + h(4)z^{-2} + h(6)z^{-3} + h(8)z^{-4}, \]

\[ = 1 + (1/2)2z^{-1} + (1/2)4z^{-2} + (1/2)6z^{-3} + (1/2)8z^{-4}, \]

\[ E_1(z) = h(1) + h(3)z^{-1} + h(5)z^{-2} + h(7)z^{-3} + h(9)z^{-4}, \]

\[ = (1/2) + (1/2)3z^{-1} + (1/2)5z^{-2} + (1/2)7z^{-3} + (1/2)9z^{-4}. \]

b) For \( g(n) = a^nU(n) \), its transfer function can be written as

\[ G(z) = \frac{1}{1 - az^{-1}} = \frac{1}{1 - a^2z^{-2}} + \frac{az^{-1}}{1 - a^2z^{-2}}. \]

By using this, we can obtain the polyphase components of \( H(z) \) as follows,

\[ E_0(z) = \frac{1}{1 - (1/4)z^{-1}} + \frac{(1/3)^4z^{-2}}{1 - (1/9)z^{-1}}, \]

\[ E_1(z) = \frac{1/2}{1 - (1/4)z^{-1}} + \frac{(1/3)^3z^{-1}}{1 - (1/9)z^{-1}}. \]

4.13. Because \( 1/(1 + az^{-1}) = (1 - az^{-1})/(1 - a^2z^{-2}) \), \( H(z) \) can be written as \( E_0(z^2) + z^{-1}E_1(z^2) \) with the polyphase components

\[ E_0(z) = \frac{a(1 - z^{-1})}{1 - a^2z^{-2}}, \quad E_1(z) = \frac{1 - a^2}{1 - a^2z^{-2}}. \]

Note that neither polyphase component is allpass (even if a is real so that \( H(z) \) is allpass).

4.14. We can rewrite

\[ H(z) = \frac{1 + 2R \cos \theta z^{-1} + R^2z^{-2}}{(1 + R^2z^{-2})^2 - 4R^2\cos^2 \theta z^{-2}}. \]
4.21.

\[ E_0(z) = 1 + 4z^{-1} + z^{-2}, \quad E_1(z) = 2 + 2z^{-1} \]

Fig. S4-21.

4.22.

a) For \( M = 3 \), we get \( m_0 = 1 \). For \( M = 4 \), \( m_0 = 3 \).

b) By direct substitution, it can be verified that \( m_0 = N \mod M \) holds true.

4.23.

a) Using Kaiser's order estimation formula for equiripple filters (3.2.32).

\[ N_p = \frac{-10\log(\delta_1 \delta_2/2) - 13}{14.6M_1(\frac{1}{M_1} - \frac{1}{2\pi})} \quad (S4.23a) \]

\[ N_i = \frac{-10\log(\delta_1 \delta_2/2) - 13}{14.6(\frac{1}{M_1} - \frac{1}{2\pi})} \quad (S4.23b) \]

From the equations, it is clear that as \( M_1 \) increases, \( N_p \) decreases but \( N_i \) will increase because \( M_1 \) occurs as a reciprocal in its denominator.

b) For the given numerical values, \( M_1 = 2, 3, 4, 5 \) are the permissible values. From the graph, we see that \( M_1 = 3 \) is the optimal value.

Fig. S4-23.
\[ g_1(z) = \sum_{n=0}^{N} g[n] z^{-2n} \quad \text{order } 2N = \text{even} \]

\[ f(2) = \frac{1}{2} 2^{-N} + \frac{1}{2} g(2) \]

\[ \frac{1}{2} 2^{-N} + \frac{1}{2} \sum_{n=0}^{N} g[n] z^{-2n} = \text{order } 2N \quad \text{(even)} \]

only odd power \( z \)

\[ F(e^{j\omega}) = \frac{1}{2} e^{-j\omega n} + \frac{1}{2} g(e^{j\omega}) \]

\[ \frac{1}{2} e^{-j\omega n} + \frac{1}{2} e^{-j\omega n} = g_R(2\omega) \quad \text{from (1)} \]

\[ F_R(\omega) = \frac{1}{2} \left( 1 + g_R(2\omega) \right) \]

\[ \frac{1}{2} \left( 1 + g_R(2\omega) \right) \]

\[ \int_{\omega_s}^{\omega_r} \frac{1 + e^{j\omega}}{2} \quad 0 \leq \omega \leq \omega_r = \omega_r \]

\[ \int_{\omega_s}^{\omega_r} \frac{1 + e^{j\omega}}{2} \quad 0 \leq \omega \leq \pi \]

\[ \omega_s = \pi - \frac{\omega_r}{2} = \pi - \omega_r \]

\[ S = \frac{\omega_r}{2}, \quad \omega_r = \frac{\omega_r}{2}, \quad \omega_s = \pi - \omega_s \]

So \( S = \frac{\omega_r}{2}, \quad \omega_r = \frac{\omega_r}{2}, \quad \omega_s = \pi - \omega_s \)

c) follow from equation (2)

d) \[ h(e^{j\omega}) = \begin{cases} 1 & \omega \in [\omega_r, \pi] \\ 0 & \text{else} \end{cases} \]

\[ \min_{\omega} F_R(\omega) = -\frac{\omega_r}{2}, \quad h(e^{j\omega}) \geq 0 \]
This clearly implies $H_1(z) = \pm (1/2)(1 - z^{-1})$.

b) They are allpass complementary.

c) They are Euclidean complementary.

d) They are doubly complementary.

4.30.

a) We have

$$F(e^{j\omega}) = (e^{-j\omega N} + e^{-j\omega N} G_R(2\omega))/2$$

hence $F_R(\omega) = (1 + G_R(2\omega))/2$.

b) We get a response identical to Fig. 4.6-4 with $\delta = 0.5\epsilon$, $\omega_p = \theta_p/2.0$ and $\omega_s = \pi - \theta_p/2.0$.

c) This can be easily verified from the definition of $F(z)$.

d) From the frequency response, it is clear that we need to ‘raise’ the response by $0.5\epsilon$, which is done by adding $0.5\epsilon$ to the central coefficient of the Type-1 filter.

4.31.

a) The eigenvalue with the largest magnitude will be $1 - \alpha \lambda_{N-1}$. Hence for stability, this should be greater than $-1$, by which we get the required condition.

b) The maximum eigenvalue-magnitude is minimized when the smallest and largest eigenvalues have equal magnitude. This means $-1 + \alpha \lambda_{N-1} = 1 - \alpha \lambda_0$, solving which we get the required condition.

4.32.

a) Computing the product, $A v_k = u_k$, we have

$$u_k(1) = 2\sin(k\pi/N + 1) - \sin(2k\pi/N + 1) = 4\sin^2(k\pi/2(N + 1))\sin(k\pi/(N + 1)).$$

$$u_k(N) = 2\sin(Nk\pi/N + 1) - \sin((N - 1)k\pi/N + 1) = 4\sin^2(k\pi/2(N + 1))\sin(Nk\pi/N + 1).$$

For all other terms, we have

$$u_k(p) = -\sin((p - 1)k\pi/N + 1) + 2\sin(pk\pi/N + 1) - \sin((p + 1)k\pi/N + 1)$$

$$= 2\sin(pk\pi/N + 1)[1 - 2\cos(k\pi/N + 1)] = 4\sin^2(k\pi/2(N + 1))\sin(pk\pi/N + 1).$$