1. It is obvious that the rank of the matrix is 2 (as the two rows and the first two columns are linearly independent). The two linearly independent rows span the row space (i.e., they form a basis for the row space), which is obviously 2 dimensional. The first two, linearly independent columns, span the column space (i.e., they form a basis for the column space), which is obviously 2 dimensional. The nullspace is obviously 1 dimensional and spanned by the vector $(0 \ 0 \ 1)^T$.

**Meyer 5.1.8**

(a) To show $||x||_1 \geq ||x||_2$, we have,

$$||x||_1^2 = \left( \sum_{i=1}^{n} |x_i| \right)^2 = \sum_{i=1}^{n} |x_i|^2 + \sum_{i \neq j} |x_i||x_j| = ||x||_2^2 + C$$

where $C \geq 0$. So $||x||_1^2 \geq ||x||_2^2$, i.e. $||x||_1 \geq ||x||_2$.

- To show $||x||_2 \geq ||x||_\infty$, we have,

$$||x||_2^2 = \sum_{i=1}^{n} |x_i|^2 \geq \max_j |x_j|^2 = ||x||_\infty^2$$

so $||x||_2 \geq ||x||_\infty$.

(b) To show $||x||_1 \leq \sqrt{n} ||x||_2$, we have,

$$||x||_1 = e^T|x| \leq ||e||_2 ||x||_2 = \sqrt{n} ||x||_2$$

where $e$ is the vector of all 1’s, and $|x|$ is the vector whose $i$th component is $|x_i|$.

- To show $||x||_2 \leq \sqrt{n} ||x||_\infty$, we have,

$$||x||_2^2 = \sum_{i=1}^{n} |x_i|^2 \leq \sum_{i=1}^{n} \max_j |x_j|^2 = n \max_j |x_j|^2 = n ||x||_\infty^2$$

so $||x||_2 \leq \sqrt{n} ||x||_\infty$.

- To show $||x||_1 \leq n ||x||_\infty$, we have,

$$||x||_1 = \sum_{i=1}^{n} |x_i| \leq \sum_{i=1}^{n} \max_j |x_j| = n \max_j |x_j| = n ||x||_\infty.$$