Reading
In addition to reviewing the course notes and past textbook reading assignments, you should carefully read and understand all of the lecture supplements and computer assignment descriptions located on the class website. You will be responsible for the material in all of these sources on the final exam.

Homework Problems

1. Vocabulary.
   Define the following: Regularized Least-Squares Problem; Maximum Likelihood Estimator; Multivariate Taylor Series Expansion; Gradient Descent Algorithm; Gauss-Newton Algorithm; Newton Algorithm; Generalized Gradient Descent Algorithm; Method of Lagrange Multipliers.

   Using nonlinear least-squares, we wish to solve the nonlinear scalar inverse problem,
   \[ y = h(x) \]
   where both \( x \) and \( y \) are real and one-dimensional. Derive the following algorithms from first principles and show that each of them is a special case of generalized gradient descent (expressly show \( Q_j \) for each of the algorithms).
   (a) Gradient Descent Algorithm.
   (b) Gauss–Newton Algorithm (from iterative re-linearization)
   (c) Newton Algorithm (from iterative quadratic approximation of the loss function).

   Now use each of the three algorithms derived in the previous problem to find the cube root of an arbitrary real-number. Code your algorithms in Matlab and use the algorithms to find the cube root of \( 1,000 \pi \) using step sizes of 1.0; 0.5; and 0.1, and initial conditions of 3,000; 100, and 0.

4. Lagrange Multipliers.
   Using the method of Lagrange multipliers minimize the nonlinear function \( 2x_1^2 + 27x_2^3 \) subject to the linear constraint \( x_1 - 9x_2 = 9 \).
5. Root finding and Functional Minimization.

The Newton-Raphson method of finding a real root, \( x_0 \in \mathbb{R} \), of a real differentiable scalar function \( f(x) \), \( f(x_0) = 0 \), is based on using the iteration,

\[
x_{j+1} = x_j - \frac{f(x_j)}{f'(x_j)},
\]

where \( f'(x) \) denotes the derivative of \( f(x) \) with respect to \( x \).

To minimize a differentiable scalar function, \( \ell(x) \), of a scalar variable \( x \), we look for roots of the derivative \( \ell'(x) \).

Show that applying the Newton (generalized gradient descent) method to minimize \( \ell(x) \) is the same as using the Newton–Raphson method to finding a root of \( \ell'(x) \) when the step-size is taken to be 1.