CAPACITY ANALYSIS OF MULTIPLE ANTENNA SYSTEMS WITH MISMATCHED CHANNEL QUANTIZATION SCHEMES

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ABSTRACT

We consider in this paper the analysis of transmit beamforming methods in multiple antenna systems with finite-rate feedback of the channel state information. We focus our attention on providing capacity analysis of a quantized MISO system over correlated fading channels with sub-optimal and mismatched channel quantizers. Two types of mismatched quantizers are investigated, which include: 1) quantizers designed with simple suboptimal criterion, and 2) quantizers whose codebooks are designed with a mismatched channel covariance matrix. We approach this problem from a source coding perspective by formulating the quantized MISO system as a general vector quantization problem with encoder side information, constrained quantization space and non-mean-squared distortion function. By utilizing the high-resolution distortion analysis of the generalized quantizer, we obtain tight lower bounds of the capacity loss of a quantized MISO system with both optimal and mismatched channel quantizers. Theoretical as well as empirical results reveal significant performance degradation of the mismatched quantizers when compared to the optimal channel quantizers. This elucidates the importance of choosing proper codebook design criterion and using correct source statistical distributions.

1. INTRODUCTION

Communication systems using multiple antennas have recently received much attention due to their promise of providing significant capacity increases. The performance of the multiple antenna systems depends heavily on the availability of the channel state information (CSI) at the transmitter (CSIT) and at the receiver (CSIR). Most of the MIMO system design and analysis adopt one of two extreme CSIT assumptions, *complete CSIT* and *no CSIT*. In this paper, we consider systems with CSI assumptions in between these extremes. We assume perfect CSIR is available at the receiver, and focus our attention on MIMO systems where CSI is conveyed from the receiver to the transmitter through a finite-rate feedback link. Recently, several interesting papers have appeared, proposing design algorithms as well as analytically quantifying the performance of the finite-rate feedback multiple antenna systems.

Most past works on the analysis of finite-rate feedback MIMO systems have adopted one of three approaches. The first is to approximate the channel quantization region corresponding to each code point based on the channel geometric property. Mukkavilli et. al. [1] derived a universal lower bound on the outage probability of quantized MISO beamforming systems with arbitrary number of transmit antennas t over i.i.d. Rayleigh fading channels. Love and et. al. [2] related the problem to that of Grassmannian line packing and provided corresponding performance bounds of multiple antenna systems with finite-rate feedback. The second approach is based on approximating the statistical distribution of the key random variable that characterizes the system performance. This approach was used by Xia et. al. in [3] and Roh et. al. in [4], where the authors analyzed the performance of MISO systems over i.i.d. Rayleigh fading channels, and obtained closed form expressions of the capacity loss (or SNR loss) in terms of feedback rate B and antenna size t. The third approach adopted by Narula et. al. in [5] is based on relating the quantization problem to the rate distortion theory, where the authors obtained an approximation of the expected loss of the received SNR due to finite rate quantization of the beamforming vectors in an MISO system. However, all these approaches are case specific and quite limited, i.i.d. channels and mainly MISO channels, and hard to extend to more complicated schemes.

In this paper, we consider the analysis of a finite-rate quantized multiple antenna system over correlated fading channels. We focus our attention on providing capacity analysis of a quantized MISO system with mismatched channel quantizers and sub-optimal channel quantization schemes. To be specific, two types of mismatched quantizers are investigated, which include: 1) quantizers that are designed with minimum mean square error (MMSE) criterion, and 2) quantizers whose codebooks are designed with a mismatched channel covariance matrix. We approach this problem from a source coding perspective by formulating the finite-rate quantized MISO system as a general vector quantization problem with encoder side information, constrained quantization space and non-mean-squared distortion function. By utilizing the high-resolution distortion analysis of the generalized vector quantizer provided in [6], we obtain tight lower bounds of the capacity loss of a quantized MISO system over correlated fading channels with both optimal and mismatched channel quantizers. Theoretical results reveal significant performance degradation of the mismatched quantizers when compared to the optimal channel quantizers. This elucidates the importance of choosing appropriate codebook design criterion and using correct source statistical distributions. Finally, numerical and simulation results are presented that confirm the accuracy of the obtained analytical results.

2. SYSTEM MODEL

We consider an MISO system with t transmit antennas, one single receive antenna, signaling through a frequency flat block fading channel. The channel impulse response $\mathbf{h}$ is assumed to be perfectly known at the receiver but partially available at the transmitter through CSI feedback. It is assumed that there exists a finite rate feedback link of B $\left(N = 2^B\right)$ bits per channel update between the transmitter and receiver. To be specific, a codebook $\mathcal{C} = \left\{\widehat{\mathbf{v}}_1, \cdots, \widehat{\mathbf{v}}_N\right\}$, which is composed of unit norm transmit beamforming vectors, is assumed known to both the receiver and the transmitter. Based on the channel realization $\mathbf{h}$, the receiver selects the

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best code point $\widehat{\mathbf{v}}$ from the codebook and sends the corresponding index back to the transmitter. At the transmitter, vector $\widehat{\mathbf{v}}$ is employed as the transmit beamforming vector, and the system channel model can be represented as

$$y = \mathbf{h}^H \cdot (\widehat{\mathbf{v}} \cdot s) + n = \|\mathbf{h}\| \cdot \langle \mathbf{v}, \widehat{\mathbf{v}} \rangle \cdot s + n \ , \quad (1)$$

where y is the received signal (scalar), $n \sim \mathcal{N}_c(0, 1)$ is the additive complex Gaussian noise with zero mean and unit variance, $\mathbf{h}^H \in \mathbb{C}^{1\times t}$ is the MISO channel response with distribution given by $\mathbf{h} \sim \mathcal{N}_c(\mathbf{0}, \mathbf{\Sigma}_h)$, and vector $\mathbf{v}$ is the channel directional vector given by $\mathbf{v} = \mathbf{h}/\|\mathbf{h}\|$. The transmitted signal s is normalized to have a power constraint given by $E\left[s^2\right] = \rho$, with ρ representing the average signal to noise ratio at each receive antenna.

The performance a finite-rate feedback MISO beamforming system can be characterized by the capacity loss C_{Loss}, which is the expectation of the instantaneous mutual information rate loss $C_L(\mathbf{h}, \widehat{\mathbf{v}})$ due to the finite rate quantization of the transmit beamforming vector. This performance metric was also used in [4] and is defined as

$$C_L(\mathbf{h}, \widehat{\mathbf{v}}) = -\log_2\left(1 - \frac{\rho \cdot \|\mathbf{h}\|^2}{1 + \rho \cdot \|\mathbf{h}\|^2} \cdot \left(1 - |\langle \mathbf{v}, \widehat{\mathbf{v}} \rangle|^2\right)\right) , \quad (2)$$

3. BACKGROUND INFORMATION

The analysis of finite-rate feedback multiple antenna systems has proven to be difficult and results available to date are limited to i.i.d. channels and mainly MISO channels. In this paper, we attempt to provide capacity analysis of both the optimal and mismatched quantizers of an MISO system over correlated fading channels. We briefly describe in this section the generalized high rate quantization theory [6] which can be used to analyze multiple antenna systems with quantized CSIT.

3.1. General Vector Quantization Framework

The multiple antenna systems with finite-rate feedback can be modeled as a generalized vector quantization problem with additional attributes such as encoder side information, constrained quantization space and non-mean-squared distortion measures. To be specific, the source variable $\mathbf{x} = (\mathbf{y}, \mathbf{z})$ is a two-vector tuple with vector $\mathbf{y} \in \mathbb{Q}$ representing the actual quantization variable of dimension k_q and $\mathbf{z} \in \mathbb{Z}$ being the additional side information of dimension k_z. The *encoder side information* $\mathbf{z}$ is available at the encoder but not at the decoder. Based on a particular source realization $\mathbf{x}$, the encoder (or the quantizer) represents vector $\mathbf{y}$ by one of the N vectors $\widehat{\mathbf{y}}_1, \widehat{\mathbf{y}}_2, \cdots, \widehat{\mathbf{y}}_N$, which form the codebook. The encoding or the quantization process is denoted as $\widehat{\mathbf{y}} = \mathcal{Q}(\mathbf{y}, \mathbf{z})$. The distortion of a finite-rate quantizer is defined as

$$D = E_{\mathbf{x}}\left[D_Q\Big(\mathbf{y}, \widehat{\mathbf{y}}\,;\,\mathbf{z}\Big)\right] , \quad (3)$$

where $D_Q\Big(\mathbf{y}, \widehat{\mathbf{y}}\,;\,\mathbf{z}\Big)$ is a general *non mean-squared distortion* function between $\mathbf{y}$ and $\widehat{\mathbf{y}}$ that is parameterized by $\mathbf{z}$. It is further assumed that function D_Q has a continuous second order derivative w.r.t. to $\mathbf{y}$.

Under high resolution assumptions, the average asymptotic distortion can be represented by the following form, which is similar to the Bennett's integral provided in [7]

$$D = 2^{-\frac{2B}{k_q}} \int_{\mathbb{Z}}\int_{\mathbb{Q}} I\big(\mathbf{y}\,;\,\mathbf{z}\,;\,\mathbb{E}_{\mathbf{z}}(\mathbf{y})\big) p(\mathbf{y}, \mathbf{z}) \lambda(\mathbf{y})^{-\frac{2}{k_q}}\ d\mathbf{y}\ d\mathbf{z}, \quad (4)$$

where $\mathbb{E}_{\mathbf{z}}(\mathbf{y})$ denotes the asymptotic projected Voronoi cell that contains $\mathbf{y}$ with side information $\mathbf{z}$. In equation (4), $\lambda(\mathbf{y})$ is a function representing the relative density of the codepoints, which is called point density, such that $\lambda(\mathbf{y})\,d\mathbf{y}$ is approximately the fraction of quantization points in a small neighborhood of $\mathbf{y}$. Function $I\big(\mathbf{y}\,;\,\mathbf{z}\,;\,\mathbb{E}\big)$ is the normalized inertia profile that represents the relative distortion of the quantizer $\mathcal{Q}$ at position $\mathbf{y}$ conditioned on side information $\mathbf{z}$ with Voronoi shape $\mathbb{E}$. Both $\lambda(\mathbf{y})$ and $I\big(\mathbf{y}\,;\,\mathbf{z}\,;\,\mathbb{E}\big)$ are the key performance determining characteristics that can be used to analyze the effects of different system parameters, such as source distribution, distortion function, quantization rate etc., on the finite-rate quantizer.

Note that if the source variable (vector) $\mathbf{y}$ is further subject to k_c *constraints* given by the vector equation $\mathbf{g}(\mathbf{y}) = 0$, the asymptotic distortion integral given by (4) is still valid under some minor modifications. In these cases, the actual degrees of freedom of the quantization variable reduce from k_q to $k_q' = k_q - k_c$, and the average asymptotic distortion decays exponentially with rate $2^{-2B/k_q'}$.

3.2. Application to Quantized MISO Systems

By employing the general framework described in Section 3.1, the finite-rate quantized MISO beamforming system can be formulated as a general fixed rate vector quantization problem.

Specifically, the source variable to be quantized is the channel directional vector $\mathbf{v}$ of $k_q = 2t$ real dimensions, and the encoder side information is the channel power $\alpha = \|\mathbf{h}\|^2$. Moreover, under the norm and phase constraints, i.e. $\mathbf{v}$ is a unit norm vector and is invariant to arbitrary phase rotation $e^{j\theta}$, the actual free dimensions of vector $\mathbf{v}$ is reduced from k_q to $k_q' = 2t - 2$. The instantaneous capacity loss due to effects of finite-rate CSI quantization is taken to be the system distortion function $D_Q(\mathbf{v}, \widehat{\mathbf{v}}\,;\,\alpha)$, which is given by the following form according to the definition given by (2)

$$D_Q(\mathbf{v}, \widehat{\mathbf{v}}\,;\,\alpha) = -\log_2\left(1 - \frac{\rho\alpha}{1 + \rho\alpha} \cdot \left(1 - \big|\langle \mathbf{v}, \widehat{\mathbf{v}} \rangle\big|^2\right)\right). \quad (5)$$

By utilizing the distortion analysis provided in [6], the normalized inertia profile of the MISO system is tightly lower bounded by

$$\widetilde{I}_{c,\,\text{opt}}(\mathbf{v}; \alpha) = \frac{\rho\alpha}{\ln 2 \cdot (1 + \rho\alpha)} \cdot \frac{(t-1)\cdot \gamma_t^{-1/(t-1)}}{t} \quad , \quad (6)$$

where γ_t is a constant coefficient equal to $\gamma_t = \pi^{t-1}/(t-1)!$. The minimal distortion of the MISO system is hence achieved by using a codebook with an optimal point density given by

$$\lambda^*(\mathbf{v}) = \beta_1(\rho, t, \mathbf{\Sigma}_h)^{-1} \cdot \left(\left(\mathbf{v}^H \mathbf{\Sigma}_h^{-1} \mathbf{v}\right)^{-(t+1)} \times {}_2F_0\left(t+1, 1;\,;\, -\frac{\rho}{\mathbf{v}^H \mathbf{\Sigma}_h^{-1} \mathbf{v}}\right)\right)^{(t-1)/t} . \quad (7)$$

where ${}_2F_0$ is the generalized hypergeometric function, and β_1 is a normalization constant that only depends on the antenna size t, channel correlation matrix $\mathbf{\Sigma}_h$ and system SNR ρ. The average system distortion (or capacity loss) of the quantized MISO system is then tightly lower bounded by

$$\widetilde{D}_{\text{c-Low}}(\mathbf{\Sigma}_h) = \frac{\rho\,(t-1)\,\beta_1(\rho, t, \mathbf{\Sigma}_h)^{t/(t-1)}}{\ln 2 \cdot |\mathbf{\Sigma}_h| \cdot \gamma_t^{t/(t-1)}} \cdot 2^{-B/(t-1)}. \quad (8)$$

4. ANALYSIS OF MISMATCHED QUANTIZERS

The asymptotic analysis provided in Section 3.2 is of MISO systems with optimal CSI quantizer, in the sense that the codebook or the encoding algorithm is designed to perfectly match the distortion function and the source distribution. However, imperfect codebook and suboptimal quantizer might be used in practical situations in order to reduce the design and encoding complexity or due to imperfect knowledge of the source distribution. We provide in this section a capacity loss analysis of the quantized MISO beamforming system when the quantizer is mismatched and suboptimal. The results further serve to demonstrate the usefulness and generality of the proposed framework in Section 3.1.

4.1. Dimensionality (Quantization Criterion) Mismatch

In this subsection, we present the analysis of a suboptimal (mismatched) quantizer that directly quantizes the CSI using the mean square error (MSE) as the distortion measure to illustrate the importance of utilizing appropriate encoding (or quantizing) algorithm in conjunction with the distortion function of interest.

For an MMSE channel quantizer, the channel state information $\mathbf{h}$ is directly quantized and the system can be viewed as a conventional vector quantization problem with the source variable having $2t$ free (real) dimensions. The corresponding distortion function of the MMSE channel quantizer is given by the following form

$$D_{\text{mis}}\big(\mathbf{h},\,\widehat{\mathbf{h}}\big) \;=\; \|\mathbf{h}-\widehat{\mathbf{h}}\|^2 \;. \tag{9}$$

At the transmitter, the unit norm beamforming vector $\widehat{\mathbf{v}}$ is obtained by normalizing the quantized channel vector $\widehat{\mathbf{h}}$, i.e. $\widehat{\mathbf{v}} = \widehat{\mathbf{h}}/\|\widehat{\mathbf{h}}\|$. Hence, the actual system distortion function (or the capacity loss) can be expressed by the following form in terms of vectors $\mathbf{h}$ and $\widehat{\mathbf{h}}$, which is given by

$$D'_{\text{Q}}\big(\mathbf{h},\,\widehat{\mathbf{h}}\big) = \log_2\left(1+\rho\cdot\|\mathbf{h}\|^2\right)-\log_2\left(1+\rho\cdot\frac{|\langle\,\mathbf{h},\,\widehat{\mathbf{h}}\rangle|^2}{\|\widehat{\mathbf{h}}\|^2}\right) \;. \tag{10}$$

It is evident from the above discussion that the MMSE channel quantizer suffers from two types of mismatches: 1) The quantizer is designed to quantize a redundant channel state information vector $\mathbf{h}$ of dimensions $2t$ instead of $2t-2$ in the optimal quantizer, which leads to a dimensionality mismatch; 2) The quantizer also uses a mismatched distortion function D_{mis} given by (9) as compared to D_{Q} given by equation (5). Due to the aforementioned suboptimality, the codebook generated by the MMSE criterion has a suboptimal point density given by

$$\lambda_{\text{mis-D}}(\mathbf{h}) = p\,(\mathbf{h})^{t/(t+1)}\cdot\left(\int p\,(\mathbf{h})^{t/(t+1)}\,d\,\mathbf{h}\right)^{-1} \;, \tag{11}$$

where $p(\mathbf{x})$ is the PDF of the MISO channel response $\mathbf{h}$. Moreover, the suboptimal MMSE quantizer also leads to a mismatched normalized inertial profile given by

$$I_{\text{mis-D}}(\mathbf{h}) = \frac{(t-1)(t!)^{1/t}\,\rho}{\ln 2\cdot(t+1)\cdot\pi\cdot(1+\rho\,\|\mathbf{h}\|^2)} \;. \tag{12}$$

By substituting equations (11) and (12) into the asymptotic distortion integration given by (4), we can prove that the asymptotic distortion lower bound of a mismatched MMSE channel quantizer can be represented by the following closed form expression

$$\widetilde{D}_{\text{mis-D}}\,(\mathbf{\Sigma}_{\text{h}}) = \frac{(t-1)\cdot\left[\,t\,!\,\right]^{1/t}\cdot\left[(t+1)/t\right]^{t}\cdot\beta_2\,(\rho,\,t,\,\mathbf{\Sigma}_{\text{h}})}{\ln 2\cdot t\cdot|\mathbf{\Sigma}_{\text{h}}|^{(t-1)/(t^2+t)}}\cdot 2^{-B/t}, \tag{13}$$

where $\beta_6\,(\rho,\,t,\,\mathbf{\Sigma}_{\text{h}})$ is a constant coefficient given by

$$\beta_2\,(\rho,\,t,\,\mathbf{\Sigma}_{\text{h}}) = -\frac{t}{t+1}\sum_{i=1}^{t}\left(\lambda_{\text{h},\,i}\prod_{k\neq i}\left(1-\frac{\lambda_{\text{h},\,k}}{\lambda_{\text{h},\,i}}\right)\right)^{-1} \cdot\exp\left(\frac{t}{\rho\,(t+1)\lambda_{\text{h},\,i}}\right)\cdot E_{\text{i}}\left(-\frac{t}{\rho\,(t+1)\lambda_{\text{h},\,i}}\right). \tag{14}$$

where $E_{\text{i}}(\cdot)$ is the exponential integral function.

It can be observed from (13) that the system distortion of the mismatched MMSE channel quantizer decays slower (with slope $-1/t$ in the exponent) than that of the optimal quantizer (with slope $-1/(t-1)\big)$. This is a significant system performance degradation especially when the size of the antenna array is small, which means it is very important to choose an appropriate CSI quantization scheme as well as a proper distortion metric function.

4.2. Source Distribution Mismatch (or Point Density Mismatch)

For the correlated MISO channels, the channel distribution depends on the covariance matrix $\mathbf{\Sigma}_{\text{h}}$, which needs to be estimated and is subject to estimation error. Moreover, it is also practically infeasible to redesign codebooks for every $\mathbf{\Sigma}_{\text{h}}$ and use it adaptively. Therefore, in practical situations, only very limited codebooks are available and they are designed with mismatched channel covariance matrix $\mathbf{\Sigma}_{\text{h}}^{\text{m}}$, which will cause performance degradation.

Based on the mismatched covariance matrix $\mathbf{\Sigma}_{\text{h}}^{\text{m}}$, a sub-optimal codebook is generated with the mismatched point density given by, from equation (7),

$$\lambda_{\text{mis-P}}\,(\mathbf{v}) = \beta_1\,(\rho,\,t,\,\mathbf{\Sigma}_{\text{h}}^{\text{m}})^{-1}\left(\left(\mathbf{v}^H\,(\mathbf{\Sigma}_{\text{h}}^{\text{m}})^{-1}\,\mathbf{v}\right)^{-(t+1)} {}_2F_0\left(t+1,\,1;\,;\,-\frac{\rho}{\mathbf{v}^H\,(\mathbf{\Sigma}_{\text{h}}^{\text{m}})^{-1}\,\mathbf{v}}\right)\right)^{(t-1)/t} \;. \tag{15}$$

By substituting the mismatched point density λ_{mis} given by (15) into the distortion integral (4), the system distortion lower bound of the covariance-mismatched quantizer can be obtained as,

$$\widetilde{D}_{\text{mis-P}} = \left(\int \widetilde{I}_{\text{c, opt}}(\mathbf{v};\alpha)\cdot p(\mathbf{h})\cdot\lambda_{\text{mis-P}}\,(\mathbf{v})^{-\frac{1}{t-1}}\;d\,\mathbf{h}\right)\cdot 2^{-\frac{B}{t-1}}. \tag{16}$$

As a special case, if the codebook designed for i.i.d. MISO channels is used for correlated MISO systems[1], i.e. $\mathbf{\Sigma}_{\text{h}}^{\text{m}} = I_t$, the mismatched point density $\lambda_{\text{mis}}(\mathbf{v})$ is uniform and the asymptotic distortion of the mismatched quantizer can be obtained by the following analytical closed form expression after some manipulations

$$\widetilde{D}_{\text{mis-P}}\,(\mathbf{\Sigma}_{\text{h}}) = \frac{(t-1)\cdot\beta_3\,(\rho,\,\mathbf{\Sigma}_{\text{h}})}{\ln 2\cdot t}\cdot 2^{-B/(t-1)} \;, \tag{17}$$

where the constant coefficient $\beta_5\,(\rho,\,\mathbf{\Sigma}_{\text{h}})$ is given by

$$\beta_3\,(\rho,\,\mathbf{\Sigma}_{\text{h}}) = \;1+\sum_{i=1}^{t}\left(\rho\,\lambda_{\text{h},\,i}\prod_{j\neq i}\left(1-\frac{\lambda_{\text{h},\,j}}{\lambda_{\text{h},\,i}}\right)\right)^{-1} \cdot\exp\left(\frac{1}{\rho\,\lambda_{\text{h},\,i}}\right)\cdot E_{\text{i}}\left(\frac{-1}{\rho\,\lambda_{\text{h},\,i}}\right) \;. \tag{18}$$

4.3. Comparison With Other Quantizers

In order to understand how the mismatched channel covariance matrix $\left(\mathbf{\Sigma}_{\text{h}}^{\text{m}} = I_t\right)$ affects the MISO system performance, a distortion comparison between optimal and mismatched quantizers under both correlated and i.i.d. fading channels is formed. By utilizing the concavity property of function $\beta_3(\rho,\mathbf{\Sigma}_{\text{h}})$ w.r.t. matrix $\mathbf{\Sigma}_{\text{h}}$, it can be proved that $\widetilde{D}_{\text{mis-P-Low, 1}}\,(\mathbf{\Sigma}_{\text{h}})$ satisfies the following inequality

$$\widetilde{D}_{\text{c-Low}}\,(\mathbf{\Sigma}_{\text{h}}) \leq \widetilde{D}_{\text{mis-P}}\,(\mathbf{\Sigma}_{\text{h}}) \leq \widetilde{D}_{\text{c-Low}}\big(I_t\big) \;. \tag{19}$$

Moreover, it can also be shown that the mismatched system distortion $\widetilde{D}_{\text{mis-P}}\,(\mathbf{\Sigma}_{\text{h}})$ converges to the distortion of i.i.d. MISO channels with optimal quantizers in high-SNR regimes,

$$\widetilde{D}_{\text{mis-P}}\,(\mathbf{\Sigma}_{\text{h}})\Big|_{\rho\to\infty} = \widetilde{D}_{\text{c-Low}}^{\text{H-snr}}\big(I_t\big) = \frac{t-1}{\ln 2\cdot t}\cdot 2^{-B/(t-1)} \;. \tag{20}$$

[1]This can be also viewed as the case where the channel covariance matrix is completely unavailable at both the transmitter and the receiver.

This means that: 1) The capacity loss of a correlated MISO channel by using the mismatched quantizer is larger than that of the optimal quantizer, but still less than the loss of an uncorrelated MISO channel even with optimal codebook. 2) The performance of the mismatched quantizer is dominated by its suboptimal codebook, and does not depend on the channel correlations in high-SNR regimes.

5. NUMERICAL AND SIMULATION RESULTS

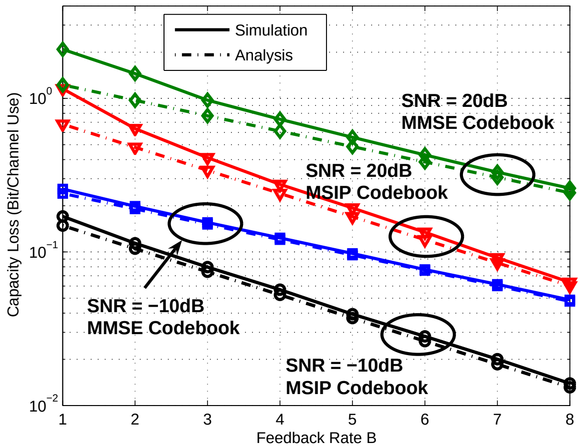

Fig. 1. Capacity loss of a 3×1 MISO system versus CSI feedback rate B using different channel quantizers (codebooks)

We plot in Fig. 1 the capacity loss due to the finite-rate quantization of the CSI versus feedback rate B for a 3×1 MISO system over i.i.d. Rayleigh fading channels with different system SNRs at $\rho = -10$, and 20 dB, respectively. Codebooks are designed by using both the optimal mean squared weighted inner-product (MSwIP) criterion proposed in [4] and the simple MMSE criterion mentioned in Section 4.1. The analytical evaluations of the system distortion lower bound $D_{\text{c-Low}}$ provide by (8) and the mismatched distortion $\widetilde{D}_{\text{mis-D}}$ provided by (13) are also included in the plot for comparisons. It can be observed from the plot that the system performance is significantly degraded by the mismatched quantizer, especially for systems with small antenna size. Moreover, the proposed distortion analysis is tight and predicts very well the actual system capacity loss obtained from Monte Carlo Simulations.

We demonstrate in Fig. 2 the system capacity loss of the mismatched quantizer with codebook designed for a 3×1 i.i.d. MISO channel but used in a correlated fading environment with SNR $\rho = 20dB$. The spatially correlated channel is simulated by the correlation model in [8]: A linear antenna array with antenna spacing of half wavelength, uniform angular-spread in $[-30°, 30°]$ and angle of arrival $\phi = 0°$. For comparison purpose, the distortion of the optimal quantizer over correlated MISO channel with $D/\lambda = 0.3$ is also included in the plot. Both the optimal and the mismatched codebooks are generated by the MSwIP criterion. It can be observed that the capacity loss of the mismatched quantizer is significantly worse than that of the optimal quantizers with optimal designed codebooks. Furthermore, it does not depend on the channel correlation in high SNR regimes and converges to that of i.i.d. channels.

6. CONCLUSION

In this paper, we provide a capacity analysis of a quantized MISO system over correlated fading channels with sub-optimal and mismatched channel quantizers. Two types of mismatched quantizers

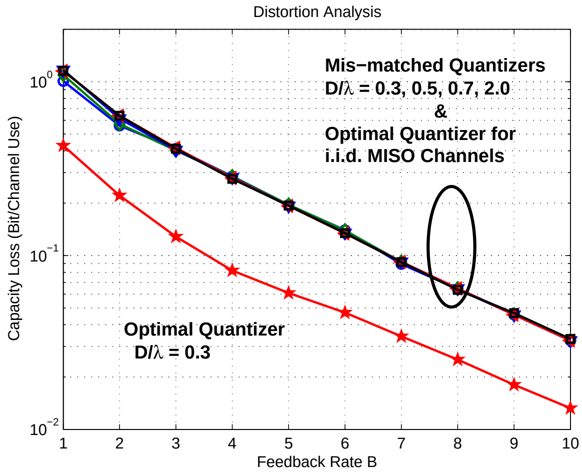

Fig. 2. Capacity loss of mismatched MISO quantizers with codebook designed for i.i.d. channels used in correlated fading channels

are investigated, which include: 1) quantizers designed with suboptimal MMSE criterion, and 2) quantizers whose codebooks are designed with a mismatched channel covariance matrix. We approach this problem from a source coding perspective, and the finite-rate quantized MISO system is first formulated as a general vector quantization problem with encoder side information, constrained quantization space and non-mean-squared distortion function. By utilizing the high-resolution distortion analysis of the generalized quantizer, we obtain tight lower bounds of the capacity loss of a quantized MISO system with both optimal and mismatched channel quantizers. Analytical results reveal significant performance degradation of the mismatched quantizers when compared to the optimal channel quantizers, especially for systems with small antenna array size and high channel correlations. This elucidates the importance of choosing appropriate codebook design criterion and using correct source statistical distributions. Finally, numerical and simulation results are presented that confirm the accuracy of the obtained analytical results.

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