RA/T Spreading Code Scheme for Centralized DS/SSMA Packet Radio Networks

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Abstract

We address an issue of channel sharing among users by using a random assignment/transmitter-oriented code scheme which permits the contention mode only in the transmission of a header. The reception of the data packet may be blocked due to limited number of correlators so that this effect is taken into account in our analysis. We also consider an acknowledgement scheme to notify whether the header is correctly detected, which aims at reducing the interference caused by unwanted data transmission. It is shown that about half reduction in receiver complexity is allowed by choosing a proper number of codes without losing performance quality.

1. Introduction

Most of direct-sequence/spread-spectrum multiple-access (DS/SSMA) packet radio networks are usually allowed to change their spreading code sequences while transmitting. If so, system throughput and complexity are largely affected by selection of spreading code sequences. In spread-spectrum networks, transmission protocol as a rule of determining such selection is specified by a number of factors, such as transmitter, intended receiver, transmission time, and priority of message, etc. Up to now, there are several studies to address this issue, but a few results [1], [2] have been reported to provide detailed analysis of performance.

This paper proposes a random assignment/transmitter-oriented (RA/T) code scheme for centralized DS/SSMA packet radio networks. Here RA/T refers to selection of two spreading codes to be used for transmission of the header and data portion of a packet. When a terminal is ready to send a packet to a central node, it chooses randomly one out of L spreading codes [2] for transmission of the header where L is considerably smaller than the number K of users. The data packet is transmitted using a distinct spreading code to avoid collision among contending packets. In the RA/T code scheme, correct detection of the header mainly determines system throughput, which enables us to continuously process the data packet by switching to one of G programmable matched-filters (correlators). But if we consider G much less than K to reduce system complexity, some of data packets may be blocked even though their preceding headers are correctly detected. System throughput is then affected by a complicated function of detection performance at the physical level and channel activity at the link level.

For evaluation of system throughput, we obtain an approximation to the probability of minipacket success by taking into account the bit-to-bit error dependence within a minipacket [3], [4], and model the network state as a Markov chain. At the link level, we account for a collision event caused by the primary user interference and a blocking event due to the limited number of correlators in deriving the state transition probability of the Markov chain. At the physical level, the effects of the secondary user interference and error-correction coding are considered when we evaluate the probability of minipacket success. Thus, system throughput normalized by a code rate and its bandwidth shows the performance of the RA/T code scheme which reflects the characteristics of physical level and also the complexity of central node.

2. System Model and RA/T Code Scheme

There are K potential user terminals communicating with a single central node in a centralized DS/SSMA packet radio network. A packet is divided into M minipackets, each of which contains l coded bits, and the first one serves as the header including a source address. We call it the header minipacket (HMP), and if l is large compared to the header length, then it may contain the data portion of a packet. The remaining $M-1$ minipackets convey a real data, which is called the data minipacket (DMP). The packet transmission is allowed every slot whose interval is equal to one minipacket length, so the network is operated in a slotted manner.

For the RA/T code scheme, every user terminal shares L spreading codes $\{c_j^{ra}\}_{j=1}^{L}$ for transmission of HMPs, which are referred to as RA (random assignment) codes. However, a near-orthogonal transmitting code c_k^t is assigned to the k-th terminal only for transmission of its DMPs, which allows to avoid collision with other HMPs and DMPs using different spreading codes. Hence we require K distinct transmitting codes for transmission of DMPs in which at most G of them can be dynamically selected depending on correct detection and decoding of their preceding HMPs. Such selected (maximum) G transmitting codes form a time-varying subset $\{c_{k_g}^t\}_{g=1}^{G}$ for some $k_g \in \{1, 2, \ldots, K\}$, namely,

$$c_{k_g}^t \in \{c_1^t, c_2^t, \ldots, c_K^t\}, \qquad c_{k_g}^t \neq c_{k_{g'}}^t \quad \text{if } g \neq g'.$$

In this situation, we need some type of coordination between surrounding K terminals and the central node to properly determine the subset $\{c_{k_g}^t\}_{g=1}^{G}$ every slot. For this, the central node should have a list of the spreading codes $\{c_j^{ra}\}_{j=1}^{L}$ and $\{c_k^t\}_{k=1}^{K}$ for reception of HMPs and DMPs, respectively.

The following procedure of transmitting a particular k-th terminal's packet illustrates such coordination, i.e., the operations of transmission protocol and central node.

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1. The central node monitors L RA code channels at the same time for reception of HMPs.
2. The k-th terminal randomly chooses one from the code set $\{c_j^{ra}\}_{j=1}^{L}$ with equal probability $1/L$ for transmission of its HMP, and then sends its DMPs using the transmitting code c_k^t.
3. The central node detects the k-th user's HMP, sets to c_k^t one of programmable matched-filters available by decoding the source address in HMP, and then demodulate the subsequent $M-1$ DMPs.

3. Probability of Minipacket Success

For coherent BPSK, decision statistic of the first user at the correlation receiver can be written as [4]

$$Z_1 = N + \sum_{j=2}^{J} MAI_j \tag{1}$$

where Z_1 is normalized with respect to the chip time T_c ($N = T_b/T_c$) with equal received power $P = 2$ for all J active users. If the MAI is approximately Gaussian when conditioned on the phases $\{\phi_j\}$ and the delays $\{\tau_j\}$ ($j = 2, 3, \ldots, J$), then the conditional probability of data bit error is approximated to

$$p_e = Q\left[\frac{N}{\sqrt{\Psi}}\right] \tag{2}$$

with Q given by $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-u^2/2}du$. In the above, the variance Ψ of the MAI is modeled as a function of random variables $\{\phi_j\}$ and $\{\tau_j\}$ ($j = 2, 3, \ldots, J$) to account for the bit-to-bit error dependence.

For a DS/SSMA system employing t error-correction coding, the probability of minipacket success at the central node can be accurately approximated to [3]

$$P_s(J) = \mathbf{E}\left[g\left(Q\left[\frac{N}{\sqrt{\Psi}}\right]; l, t\right)\right] \tag{3}$$

$$= \int_0^\infty g\left(Q\left[\frac{N}{\sqrt{\psi}}\right]; l, t\right) f_\Psi(\psi)\, d\psi \tag{4}$$

where $f_\Psi(\psi)$ denotes the probability density function of Ψ and

$$g(p_e; l, t) = \sum_{i=0}^{t} \binom{l}{i} p_e^i (1-p_e)^{l-i}. \tag{5}$$

Applying the simple, accurate approximation of [4] to (4) yields

$$\begin{aligned}\hat{P}_s(J) &= \frac{2}{3} g\left(Q\left[\frac{N}{\sqrt{\mu}}\right]; l, t\right) \\ &+ \frac{1}{6} g\left(Q\left[\frac{N}{\sqrt{\mu + \sqrt{3}\sigma}}\right]; l, t\right) \\ &+ \frac{1}{6} g\left(Q\left[\frac{N}{\sqrt{\mu - \sqrt{3}\sigma}}\right]; l, t\right).\end{aligned} \tag{6}$$

In the above, we have used the mean μ and variance σ^2 of Ψ which are given by [4]

$$\mu = \frac{(J-1)N}{3} \tag{7}$$

$$\sigma^2 = (J-1)\left[N^2 \frac{23}{360} + (N-1)\left(\frac{1}{20} + \frac{J-2}{36}\right)\right] \tag{8}$$

4. Throughput Analysis

For analysis, we simply assume a variable length of packet with geometric distribution, which leads to a Markov chain model with fewer states because of the memoryless property. Then the number M of minipackets within a packet is distributed as

$$\Pr[M = m] = q(1-q)^{m-1}, \qquad m = 1, 2, \ldots \tag{9}$$

where the mean number of minipackets is given by $\bar{M} = \frac{1}{q}$. If a terminal is in idle state or transmitting the last DMP at the $(t-1)$-th slot, the terminal is assumed to generate a new packet with probability p and then send at the beginning of the t-th slot.

As central receiver operation is envisioned here, the path to successful reception of HMPs in a slot involves two events:

1. For a given RA code channel, only one terminal should be sending its HMP in the slot, otherwise the primary MAI will cause collision.
2. The HMP must contain t or less data bit errors, otherwise proper decoding will not be allowed because of the secondary MAI.

In the above, we have assumed zero capture model which yields conservative results, since it is possible for one to be captured even though two or more terminals are sending HMPs in the same RA code channel.

The operation of terminals in the system can be represented as the Markov chain with 4 states, namely, the state (H) of sending HMP, the state (D^s) of sending DMPs received, the state (D^f) of sending DMPs not received, the idle state (I). If the HMP sent by a terminal is not correctly detected, or there is none of the matched-filters available to receive the DMPs after proper decoding of the HMP, the terminal is in the state D^f by sending the DMPs not received.

Each of K terminals in the network is considered to be identical and independent, which enable us to model the network state as the Markov chain with three state variables $z_t = (h_t, d_t^s, d_t^f)$ during the t-th slot. Here h_t, d_t^s, and d_t^f indicate the number of terminals belonging to the states H, D^s, and D^f, respectively. The state space of z_t is

$$\begin{aligned}\mathbf{Z} = \{&(n_1, n_2, n_3) : n_1 \geq 0,\ 0 \leq n_2 \leq G,\ n_3 \geq 0, \\ &\text{and } n_1 + n_2 + n_3 \leq K\}\end{aligned} \tag{10}$$

where the number of states is given by $|\mathbf{Z}| = \eta(K) - \eta(K - G - 1)$ for a function $\eta(x) = 1/6x^3 + x^2 + 1/6x + 1$.

Let us define $P(z_t|z_{t-1})$ by the probability of transition from state z_{t-1} to state z_t, that is,

$$\begin{aligned}P(z_t|z_{t-1}) &= \Pr\{z_t = (h_t, d_t^s, d_t^f) \mid z_{t-1} \\ &= (h_{t-1}, d_{t-1}^s, d_{t-1}^f)\}.\end{aligned} \tag{11}$$

First, as the conditional for derivation of $P(z_t|z_{t-1})$, consider the event that i of h_{t-1} terminals in the state H, j of d_{t-1}^s in the state D^s, and k of d_{t-1}^f in the state D^f enter into the state I at the beginning of the t-th slot by ending their transmissions. For such event denoted by $C_{i,j,k}$, we find that

$$\Pr[C_{i,j,k}] = B(i; h_{t-1}, q)\, B(j; d_{t-1}^s, q)\, B(k; d_{t-1}^f, q) \tag{12}$$

where $B(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$ and we have used the memoryless property of geometric distribution. Conditioned on the event $C_{i,j,k}$, the transition probability $P(z_t|z_{t-1})$ can be related to

$$P(z_t|z_{t-1}) = \sum_{i=0}^{h_{t-1}} \sum_{j=0}^{d_{t-1}^s} \sum_{k=0}^{d_{t-1}^f} \Pr[C_{i,j,k}] \Pr[\Gamma | C_{i,j,k}] \tag{13}$$

where Γ denotes the event of making transition from z_{t-1} to z_t.

When we look into the network at the beginning of the t-th slot, there are $K_H = h_{t-1} - i$ terminals in the state H and $K_I = K - (h_{t-1} + d_{t-1}^s + d_{t-1}^f) + (i + j + k)$ terminals in the state I. In order to make the event Γ occur, we require two conditions, i.e., i) h_t of K_I terminals have to send their HMPs, ii) given K_H terminals, $s = d_t^s - (d_{t-1}^s - j)$ of them should enter into the state D^s while $f = d_t^f - (d_{t-1}^f - k)$ enter into the state D^f. Since the K_H terminals must enter into either D^s or D^f, we have the relation $K_H = s + f$. Because of the independence of i) and ii), the probability of both events' occurrence at the same time is given by the product of each probabilities where the first one is simply evaluated as $B(h_t; K_I, p)$. To evaluate the probability of occurrence of ii), we define $P_1(s|h_{t-1}, K_H, E_t)$ by the probability that s of K_H terminals enter into the state D^s, given h_{t-1}, K_H, and E_t matched-filters available in the t-th slot. Here E_t is equal to $G - (d_{t-1}^s - j)$ under the above conditions.

The probability $P_1(s|h_{t-1}, K_H, E_t)$ can be derived as

$$P_1(s|h_{t-1}, K_H, E_t) = \begin{cases} P_1(s|h_{t-1}, K_H) \\ \qquad \text{if } s < \min(K_H, E_t), \\ \displaystyle\sum_{n=\min(K_H, E_t)}^{K_H} P_1(n|h_{t-1}, K_H) \\ \qquad \text{if } s = \min(K_H, E_t) \end{cases} \tag{14}$$

where for $K_H \le E_t$, we have

$$\begin{aligned} P_1(s|h_{t-1}, K_H) &= \left(\frac{1}{L}\right)^{h_{t-1}} \sum_{w=s}^{K_H} \binom{K_H}{w} \binom{L}{w} w! \\ &\cdot B(s; w, P_s^H) \sum_{r=0}^{K_H - w} (-1)^r \binom{K_H - w}{r} \\ &\cdot \binom{L-w}{r} r! \, (L - w - r)^{h_{t-1} - w - r}. \end{aligned} \tag{15}$$

In the above, P_s^H stands for the probability that the HMP is correctly detected in the presence of the secondary MAI caused by the $(h_{t-1} + d_{t-1}^s + d_{t-1}^f - 1)$ interfering packets. Under the random sequence model, we can apply (6) to obtain the approximation

$$P_s^H \approx \hat{P}_s(h_{t-1} + d_{t-1}^s + d_{t-1}^f). \tag{16}$$

Therefore, applying the above results to (13) yields

$$\begin{aligned} P(z_t|z_{t-1}) &= \sum_{(i,j,k) \in \Omega(z_{t-1}, z_t)} \Pr[C_{i,j,k}] \, B(h_t; K_I, p) \\ &\cdot P_1(s|h_{t-1}, K_H, E_t) \end{aligned} \tag{17}$$

where $\Omega(z_{t-1}, z_t) = \{(i,j,k) : 0 \le i \le h_{t-1}, 0 \le j \le d_{t-1}^s, 0 \le k \le d_{t-1}^f, \text{ and } K_H = s + f\}$.

Let us denote the transition matrix of the Markov chain by $\mathbf{P} = [P(z_t|z_{t-1}) : z_{t-1}, z_t \in \mathbf{Z}]$, then the steady-state probability distribution $\{\pi(z) : z \in \mathbf{Z}\}$ can be derived by solving the formula

$$\pi = \pi \mathbf{P}, \qquad \sum_{z \in \mathbf{Z}} \pi(z) = 1 \tag{18}$$

with the row vector $\pi = [\pi(z) : z \in \mathbf{Z}]$.

Now, by using the probability distribution $\{\pi(z)\}$, we can evaluate throughput for the DS/SSMA packet radio network that employs the RA/T code scheme. Here throughput is defined as the mean number of successful minipackets per slot. When the network is in the steady-state $z = (h, d^s, d^f)$, there is a contribution to throughput by some of the h HMPs whose entire packets are correctly decoded and some of the d^s DMPs which give rise to t or less data bit errors.

First, we define $\beta_h(z)$ by the mean number of such HMPs, that is, $\beta_h(z) = \mathbf{E}\{V\}$ where $\mathbf{E}$ denotes an expectation and V is the corresponding random variable. Then the random variables I and J are introduced to represent the numbers of idle terminals from the previous states H and D^s, respectively. Conditioned on the event $\{I = i, J = j\}$, $\beta_h(z)$ can be rewritten by

$$\begin{aligned} \beta_h(z) &= \mathbf{E}\{\mathbf{E}\{V = v \mid I = i, J = j\}\} \\ &= \sum_{i=0}^{h} \sum_{j=0}^{d^s} p_I(i) \, p_J(j) \sum_{v=0}^{h} v p_V(v | I = i, J = j). \end{aligned} \tag{19}$$

In the above, the probability density functions $p_I(i)$ and $p_J(j)$ are given by $B(i; h, q)$ and $B(j; d^s, q)$ because the packet length has geometric distribution. The conditional probability density function $p_V(v|I = i, J = j)$ is shown to be

$$p_V(v|I = i, J = j) = \sum_{(s,c) \in \Omega(v)} P_2(s, c|h, K_H, E) \tag{20}$$

where $K_H = h - i$, $E = G - (d^s - j)$, $\Omega(v) = \{(s,c) : 0 \le s \le K_H, 0 \le c \le i, \text{ and } v = s + c\}$, and

$$P_2(s, c|h, K_H, E) = \begin{cases} P_2(s, c|h, K_H) \\ \qquad \text{if } s < \min(K_H, E), \\ \displaystyle\sum_{n=\min(K_H, E)}^{K_H} P_2(n, c|h, K_H) \\ \qquad \text{if } s = \min(K_H, E). \end{cases} \tag{21}$$

With $K_H \le E$, we obtain that

$$\begin{aligned} &P_2(s, c|h, K_H) \\ &= \left(\frac{1}{L}\right)^h \sum_{w=s}^{K_H} \sum_{g=c}^{i} \binom{K_H}{w} \binom{L}{w} w! \binom{i}{g} \binom{L-w}{g} g! \\ &\cdot B(s; w, P_s^H) \, B(c; g, P_s^H) \sum_{r=0}^{h-w-g} (-1)^r \binom{h-w-g}{r} \\ &\cdot \binom{L-w-g}{r} r! \, (L - w - g - r)^{h-w-g-r} \end{aligned} \tag{22}$$

where $P_s^H \approx \hat{P}_s(h + d^s + d^f)$. Substituting them into (19) gives

$$\begin{aligned} \beta_h(z) &= \sum_{i=0}^{h} \sum_{j=0}^{d^s} B(i; h, q) \, B(j; d^s, q) \\ &\cdot \sum_{s=0}^{K_H} \sum_{c=0}^{i} (s + c) P_2(s, c|h, K_H, E). \end{aligned} \tag{23}$$

Next, if we define $\beta_d(z)$ by the mean number of successful DMPs, then we have

$$\beta_d(z) = d^s P_s^D. \quad (24)$$

Here P_s^D, the probability of DMP success, is well approximated to $\hat{P}_s(h + d^s + d^f)$.

Conditioned on the steady-state z, the normalized throughput $\bar{\beta}(z)$ with respect to the code rate and bandwidth expansion has the expression

$$\bar{\beta}(z) = \frac{1}{N}\left[r_h\beta_h(z) + r_d\beta_d(z)\right]. \quad (25)$$

The r_h and r_d indicate the code rates associated with the HMP and DMP, respectively. In case of $r_h \neq r_d$, if we are using block codes for forward error correction, t in (6) needs to be replaced by the corresponding values from the Varsharmov-Gilbert lower bound [5]

$$r \geq 1 - H\left(\frac{d_{min} - 2}{l}\right) \quad (26)$$

where $H(x) = -x\log_2 x - (1-x)\log_2(1-x)$ and $d_{min} \geq 2t+1$. Finally, taking the average with respect to z yields

$$\bar{\beta} = \sum_{z \in \mathbf{Z}} \bar{\beta}(z)\,\pi(z). \quad (27)$$

5. RA/T with HMP Acknowledgement

We adopt an acknowledgement (ACK) scheme to reduce the secondary MAI caused by unwanted data transmission. To simplify the analysis, a terminal is assumed to attempt retransmission with the same probability p as for new transmission in idle state so that retransmission state is merged into idle state. So we take the composite traffic model with transmission probability p in which an ideal feedback channel is assumed for the ACK to ignore its effect on delay and throughput.

In the RA/T code scheme employing HMP acknowledgement, there is no need for the state D^f which indicates the terminals sending those DMPs not received. So the operation of terminals can be modeled as the Markov chain with three states such as H, $D = D^s$, and I. Similarly as in Section 4, the network state can be represented by $z_t = (h_t, d_t)$ at the beginning of the t-th slot. Here h_t and d_t denote the number of terminals in the states H and D, respectively. The state space of z_t becomes

$$\mathbf{Z} = \{(n_1, n_2) : n_1 \geq 0,\ 0 \leq n_2 \leq G, \text{ and } n_1 + n_2 \leq K\} \quad (28)$$

with $|\mathbf{Z}| = 1/2(G+1)(2K+2-G)$.

The transition probability is defined by

$$P(z_t|z_{t-1}) = \Pr\{z_t = (h_t, d_t)\,|\,z_{t-1} = (h_{t-1}, d_{t-1})\}. \quad (29)$$

Then it follows that

$$P(z_t|z_{t-1}) = \sum_{i=0}^{h_{t-1}}\sum_{j=0}^{d_{t-1}} \Pr[C_{i,j}]\,B(h_t; K_I, p) \cdot P_1(s|h_{t-1}, K_H, E_t) \quad (30)$$

where $\Pr[C_{i,j}] = B(i; h_{t-1}, q)\,B(j; d_{t-1}, q)$, $K_H = h_{t-1} - i$, $K_I = K - (h_{t-1} + d_{t-1}) + (i+j)$, and $E_t = G - (d_{t-1} - j)$. To evaluate $P_1(s|h_{t-1}, K_H, E_t)$, (16) is replaced by $P_s^H \approx \hat{P}_s(h_{t-1} + d_{t-1})$.

Based on the above results, the probability distribution $\{\pi(z) : z \in \mathbf{Z}\}$ can be derived through the formula (18). Looking at the steady-state $z = (h, d)$, we find that $\beta_h(z)$ and $\beta_d(z)$ are given by (23) and (24) with $d^s = d$ and $d^f = 0$, respectively. Then the normalized throughput $\bar{\beta}$ can be evaluated by using (25) and (27).

6. Results and Conclusions

For numerical computation, the number of coded bits is chosen to be $l = 100$ bits/minipacket with a single-error correction capability, i.e., $t = 1$, and the mean number of minipacket/packet is given by $\bar{M} = 10$, which implies 1000 bits/packet on the average.

In Figs. 1 and 2, we plot $\bar{\beta}$ versus the transmission probability p for various L and G when $K = 12$, $N = 7$, assuming the case without HMP acknowledgement. First, if we increase L up to 6 for fixed $G = 4$, then $\bar{\beta}$ becomes saturated in which most of the performance gain is achieved with only two RA codes, i.e., $L = 2$. So we may select $L = 2$ to reduce system complexity without causing any performance loss while for fixed $L = 2$, the number of matched-filters is changed from $G = 3$ to $G = 5$. Then we also observe that $\bar{\beta}$ becomes saturated near at $G = 4$, which leads to an optimum choice of $L = 2$ and $G = 4$. This implies that about half reduction in receiver complexity is allowed by employing the RA/T code scheme requiring $L + G = 6$ correlators when compared to the classical CDMA with $K = 12$ ones.

To compare the RA/T code scheme with possible ones based on classical CDMA in terms of throughput performance, we may consider a fixed assignment of the subset of $L + G$ distinct codes to K terminals [7] without RA code and HMP acknowledgement. With the optimum choice of $L = 2, G = 4$ for $K = 12$, $N = 7$, simulation results demonstrate that the RA/T code scheme outperforms the fixed assignment employing $L + G = 6$ codes in Fig. 3.

Figs. 4 and 5 show $\bar{\beta}$ versus p for the same parameters when the HMP acknowledgement is adopted. It is obvious that throughput can be greatly enhanced by using the HMP acknowledgement which allows to reduce the unwanted secondary MAI. In addition, a similar behavior is observed in view of $\bar{\beta}$ when we change the parameters L and G. So the above optimum choice is valid regardless of the HMP acknowledgement.

In reality, the performance of CDMA system is only interference limited, and operated under such optimum condition which gives a maximum normalized throughput for given system parameters. In this case, there exists a saturation behavior in view of $\bar{\beta}$ as seen in Figs. 1,2 and 4, 5 when we increase the parameters L and G. Using the RA/T code scheme, it is possible to minimize system complexity by choosing a proper L and G without losing performance quality in view of $\bar{\beta}$.

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References

[1] E. S. Sousa and J. A. Silvester, "Spreading code protocols for distributed spread-spectrum packet radio networks," *IEEE Trans. Commun.*, vol. COM-36, pp. 272-281, Mar. 1988.

[2] D. I. Kim and R. A. Scholtz, "On the performance of centralized DS-SS packet radio networks with random spreading code assignment," *IEEE Trans. Commun.*, vol. COM-43, pp. 2613-2621, Oct. 1995.

[3] R. K. Morrow and J. S. Lehnert, "Bit-to-bit dependence in slotted DS/SSMA packet system with random signature sequences," *IEEE Trans. Commun.*, vol. COM-37, pp. 1052-1061, Oct. 1989.

[4] J. M. Holtzman, "A simple, accurate method to calculate spread-spectrum multiple-access error probabilities," *IEEE Trans. Commun.*, vol. COM-40, pp. 461-464, Mar. 1992.

[5] W. W. Peterson and E. J. Weldon, *Error Correcting Codes.* Cambridge, MA: M.I.T. Press, 1972.

[6] M. Hall, Jr., *Combinatorial Theory.* 2nd Ed., NewYork: John Wiley & Sons, 1986.

[7] D. I. Kim, "Analysis of a direct-sequence CDMA mobile radio system with reduced set of code sequences," *IEEE Trans. Vehic. Technol.*, vol. VT-44, pp. 525-534, Aug. 1995.

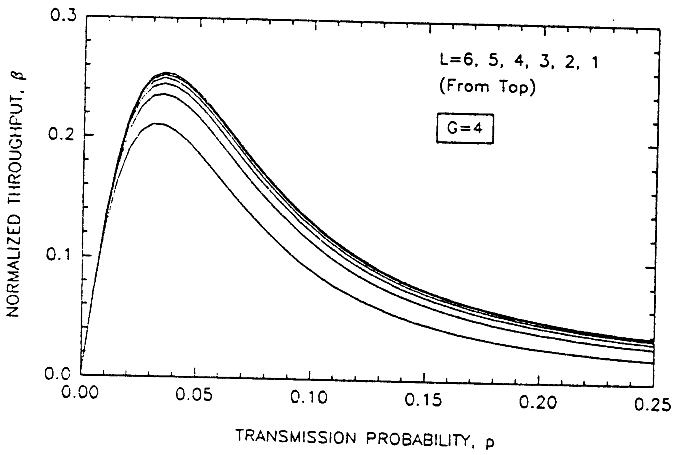

Fig. 1. Normalized throughput $\bar{\beta}$ versus p for various L when $K = 12, N = 7$.
(without HMP acknowledgement)

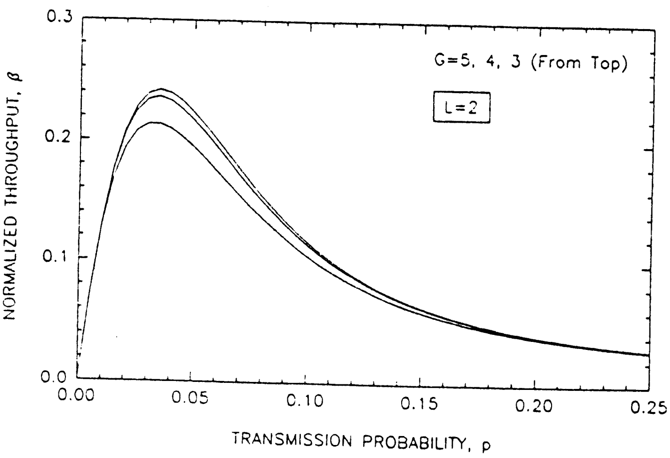

Fig. 2. Normalized throughput $\bar{\beta}$ versus p for various G when $K = 12, N = 7$.
(without HMP acknowledgement)

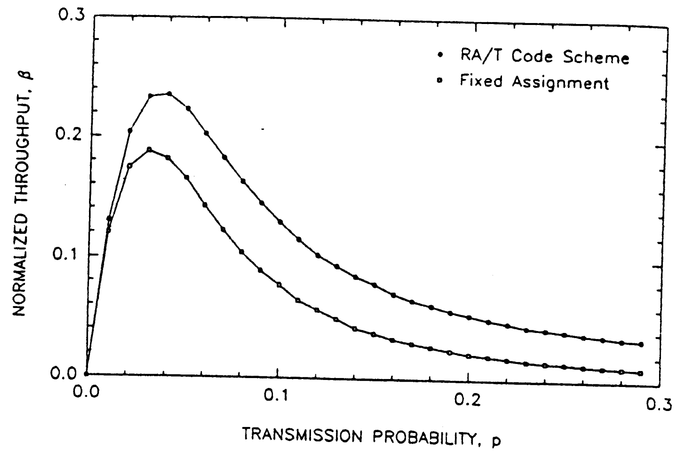

Fig. 3. Normalized throughput $\bar{\beta}$ versus p for $L = 2, G = 4$ when $K = 12, N = 7$.

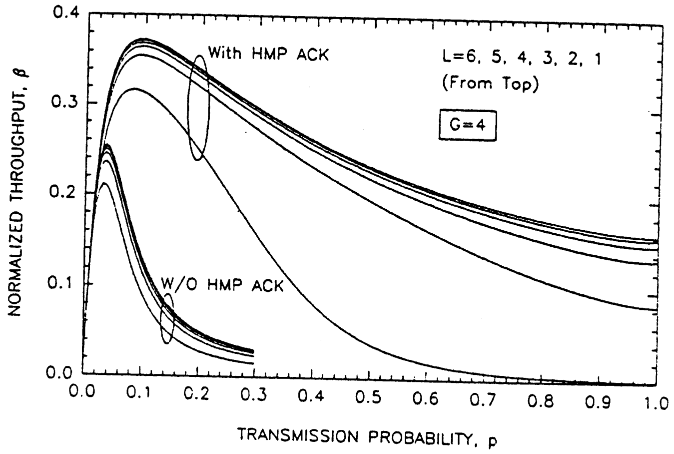

Fig. 4. Normalized throughput $\bar{\beta}$ versus p for various L when $K = 12, N = 7$.

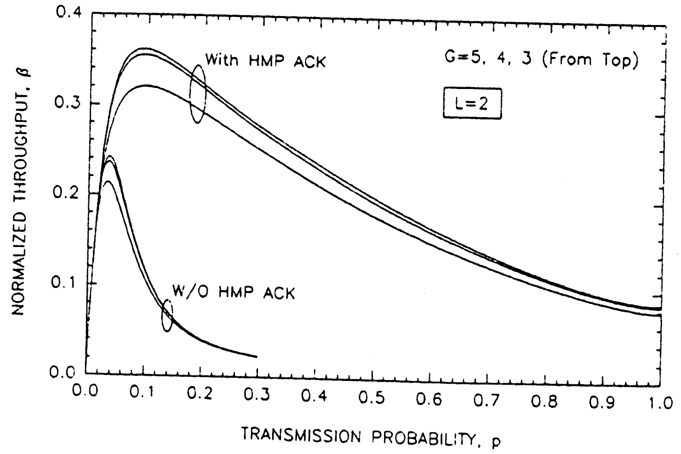

Fig. 5. Normalized throughput $\bar{\beta}$ versus p for various G when $K = 12, N = 7$.