Multiple Antenna Channels with Partial Feedback

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***Abstract*— We consider flat-fading multiple antenna channels with t transmit and r receive antennas, which is modeled by an $r \times t$ complex matrix H. The first n eigenvectors of $H^\dagger H$, where $0 \leq n \leq \min(t, r)$, are assumed to be available at the transmitter as partial spatial information of channel. A transmission method was proposed which enables better use of the multiple-input multiple-output (MIMO) channels [1]. By using the transmission scheme, the MIMO channel can be decomposed into two parts: n parallel channels and a new small coupled MIMO channel. One reasonable coding strategy is to employ conventional time-domain only code for each of the parallel channels and a space-time code for the small MIMO channel. This paper focuses on deriving the channel capacity of the multiple antenna channels employing such a coding strategy. The results show that the proposed methods lead to systems wherein the amount of feedback information can be significantly reduced with a minor sacrifice of achievable transmission rate.**

I. Introduction

Communication systems with multiple antennas at both the transmitter and the receiver have gathered much attention for high-rate transmission. There have been many studies on the information-theoretical capacity of the multiple antenna channels, immediately following the promising results by Telatar [2] and Foschini [3]. Many previous studies have focused on the following two assumptions about channel state information (CSI): the first is the case where CSI is known to both the receiver and the transmitter [2], [4]; and the second is where CSI is available only at the receiver, not at the transmitter [2], [3], [5]. We will refer to the former as *complete CSIT* and the latter as *no CSIT*[1]; these two will be used as the two references in comparing channel capacities.

We remark that there are gaps between the capacities of the two cases, in particular, when the transmit power is relatively low, or when t is greater than r. This research was motivated by a natural insight that there is a trade-off between the improvement in channel capacity and the degree of completeness of the CSI available at the transmitter. There are many applications in which there exists a feedback channel for the channel state information. The amount of channel information that is required at the transmitter can be too large to handle, since the channel has $t \times r$ number of fading parameters. Therefore, in many cases, the channel information can not be fully provided to the transmitter due to, for example, a limited transmission capacity of feedback channel, or rapid channel variation. In this study, we consider the cases where the channel information is *partially* known to the transmitter in a way that enables a reduction in the amount of the feedback information.

In designing such systems, it is important to determine what type of the channel information to feed back while minimizing the loss of channel capacity. An efficient beamforming method was proposed in which the beamforming matrix is determined from a subset of the eigenvectors of $H^\dagger H$ in some predefined way [1]; as a result, the receiver also knows the beamforming matrix. With this beamforming scheme, we introduced a new multiple antenna system concept that provides a mechanism to reduce the amount of channel feedback information. The optimum transmission strategy was investigated in the previous study.

By using the proposed transmission method, the MIMO channel can be decomposed into two parts: n parallel channels and a new small coupled MIMO channel. A reasonable coding strategy for the above mentioned channel is to employ conventional time-domain only codes for each of the parallel channels and a space-time code for the small MIMO channel. This paper focuses on deriving the channel capacity of the multiple antenna channels employing such a coding strategy. It is shown that, with this suboptimum strategy, similar performance of the optimum strategy can be achieved with the additional benefit of reduction in the amount of feedback information.

II. System Model

A. Channel Model

We consider multiple antenna systems with t antennas at the transmitter and r at the receiver. Assuming slow flat-fading, the MIMO channel is modeled by the channel matrix $H \in \mathbb{C}^{r\times t}$. That is, the channel input $x \in \mathbb{C}^t$ and the channel output $y \in \mathbb{C}^r$ have the following relationship:

$$y = Hx + \eta \tag{1}$$

where $\eta \in \mathbb{C}^r$ is the complex additive white Gaussian noise (AWGN) vector with each element being assumed *i.i.d.* complex Gaussian random variable with zero-mean and unit variance, i.e., $E\{\eta\eta^\dagger\} = I_r$, where $E\{\cdot\}$ denotes the expectation operation and I_r is the $r \times r$ identity matrix. We denote the rank of H by m. And the singular value decomposition (SVD) of H is given by $H = U\Sigma V^\dagger$, where $A^\dagger$ denotes the conjugate transpose of a matrix A; unitary

[1]CSIT: channel state information at the transmitter.

matrices $V \in \mathbb{C}^{t\times t}$ and $U \in \mathbb{C}^{r\times r}$ span the input space $\mathbb{C}^t$ and the output space $\mathbb{C}^r$, respectively; and $\Sigma \in \mathbb{R}^{r\times t}$ contains the singular values with σ_i representing the i-th singular value of H and $\sigma_1 \geq \ldots \geq \sigma_m > 0$. We impose a constraint on the transmit power, $E\{x^\dagger x\} \leq P_T$.

In this paper, we assume that in all cases perfect CSI is known to the receiver. In addition, it is assumed that the transmitter knows the first n column vectors of V, where $0 \leq n \leq m$, or the first n eigenvectors of $H^\dagger H$, as partial *spatial* information of the channel. This assumption includes the two extreme cases: i) $n = m$ is the case that the transmitter has same spatial information as in the *complete CSIT* case; and ii) $n = 0$ accounts that no spatial information is available at the transmitter as in the *no CSIT* case. This paper mainly considers the cases of $0 < n < m$; these corresponds to *partial CSIT* cases. For notational convenience, let us define $V_1 = [v_1, \ldots, v_n]$ where v_i is the i-th column vector of V, and $V_2 = [v_{n+1}, \ldots, v_t]$, i.e., $V = [V_1, V_2]$.

III. The Beamforming Method

To fully exploit potential multiplexing capability of the channel, a new and improved beamforming method was proposed in [1], [6] that also utilizes the orthogonal complement of the space spanned by V_1. A beamforming matrix $W \in \mathbb{C}^{t\times t}$ is generated as a function of V_1 in a predefined manner. Since the receiver has knowledge of V_1, the receiver is also aware of the beamforming matrix that the transmitter will use. This property enables us to conceive of a new multiple antenna system concept which is described in the next subsection. One reasonable way to generate the beamforming matrix is the following:

1) Choose $t-n$ vectors, namely, $\tilde{V}_2 = [\tilde{v}_{n+1}, \ldots, \tilde{v}_t]$, that are mutually orthogonal and also orthogonal to the space spanned by V_1, i.e.,
$$\tilde{V}_2^\dagger \tilde{V}_2 = I_{t-n}, \quad {V_1}^\dagger \tilde{V}_2 = 0 \tag{2}$$
where I_p is $p \times p$ identity matrix and 0 is $n \times (t-n)$ zero matrix.
2) Concatenate $\tilde{V}_2$ to V_1 to form a beamforming matrix $W = [V_1, \tilde{V}_2]$.

It can be easily shown that, if H is full rank, W spans the same input space as V does. The beamforming matrix W is used in transmitting the information vector $s \in \mathbb{C}^t$ in a manner similar to the use of V in the complete CSIT case. The procedure for selecting $\tilde{V}_2$ satisfying (2) can be defined in various ways, e.g., $\tilde{V}_2$ are the eigenvectors corresponding to the nonzero eigenvalues of $I_t - V_1 V_1^\dagger$. Whatever be the mechanism for generating $\tilde{V}_2$ at the transmitter, the generating mechanism is assumed to be known at the receiver so that the receiver can also independently generate $\tilde{V}_2$ and, hence, W.

A. New Multiple Antenna System Concept

With the proposed beamforming scheme, we developed a new multiple antenna system concept that can potentially lead to a reduction in the amount of channel feedback information. It involves

1) Based on W, calculation for optimal power allocation over transmit symbols is performed at the receiver.
2) The power allocation result is provided to the transmitter as additional CSIT.

The first step above will be described in detail in the next Section. Two approaches are considered, each of which results in t and $n+1$ real values, respectively. These values are bounded between 0 and 1, and sum up to be 1. Then, the total channel feedback information is n t-dimensional complex vectors, i.e., V_1, plus t (for the first approach) or $n+1$ (for the second approach) real values in $[0, 1]$. Thus, in most systems, in particular, when the number of transmit antennas t is large, the amount of feedback information can be significantly reduced. This paper focuses on the later case, and the former was discussed in [1].

B. Channel Decomposition

Now, we will show that, by using the proposed beamforming method, the original MIMO channel is decomposed into two parts. The transmitted signal x is given by
$$x = Ws = [V_1, \tilde{V}_2] \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = V_1 s_1 + \tilde{V}_2 s_2$$
where $s = [s_1, s_2]^T \in \mathbb{C}^t$, $s_1 \in \mathbb{C}^n$, and $s_2 \in \mathbb{C}^{t-n}$.

The receiver pre-multiplies the received signal $y = Hx + \eta$ by $U^\dagger$ to have $\tilde{y} = U^\dagger y$. Using the partitioned matrices of compatible sizes to $W = [V_1, \tilde{V}_2]$, $\tilde{y}$ can be written as follows:
$$\tilde{y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 V_2^\dagger \tilde{V}_2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \tilde{\eta}_1 \\ \tilde{\eta}_2 \end{bmatrix} \tag{3}$$
where $\tilde{y}_1 \in \mathbb{C}^n$, $\tilde{y}_2 \in \mathbb{C}^{r-n}$, diagonal matrices $\Sigma_1 \in \mathbb{R}^{n\times n}$ and $\Sigma_2 \in \mathbb{R}^{(r-n)\times(t-n)}$ contain $\sigma_1, \ldots, \sigma_n$ and $\sigma_{n+1}, \ldots, \sigma_m$, respectively, and zero matrices are of suitable size. Equation (3) results from the facts that $V_1^\dagger V_1 = I_n, V_1^\dagger \tilde{V}_2 = 0$, and $V_2^\dagger V_1 = 0$.

We can see that the MIMO channel has been decomposed into n non-interfering parallel channels and a new coupled MIMO channel with a channel matrix $H_2 = \Sigma_2 V_2^\dagger \tilde{V}_2$ in $\mathbb{C}^{(r-n)\times(t-n)}$. That is,
$$\tilde{y}_1 = \Sigma_1 s_1 + \tilde{\eta}_1 \tag{4}$$
$$\tilde{y}_2 = H_2 s_2 + \tilde{\eta}_2 \tag{5}$$
We will refer to the first channel of (4) as the Σ_1 channel, and the second channel of (5) as the H_2 channel. Note that the covariance of $\tilde{\eta} = U^\dagger \eta = [\tilde{\eta}_1, \tilde{\eta}_2]^T$ is unchanged as $E\{\tilde{\eta}\tilde{\eta}^\dagger\} = I_r$. An interesting property about the singular values of the channel matrix is summarized in the following Lemma.

Lemma 1: The singular values of the channel matrix $H_2 = \Sigma_2 V_2^\dagger \tilde{V}_2$ is preserved as $\mathrm{diag}(\Sigma_2)$.

Proof: See Appendix of [6]. ∎

By the following Lemma, we show that the mutual information $I(x; y)$ is preserved with the linear operations $x = \tilde{V}s$ and $\tilde{y} = U^\dagger y$. Furthermore, $I(x; y)$ can be given by the sum of the mutual information expressions for two decomposed channels.

Lemma 2: For a given channel realization H, the mutual information between the input and the output of the MIMO channel can be expressed as

$$\begin{aligned} I(x;y) &= I(s;\tilde{y}) && (6)\\ &= I(s_1;\tilde{y}_1) + I(s_2;\tilde{y}_2). && (7) \end{aligned}$$

Proof: See Appendix of [6]. ∎

IV. MIMO Channel with Partial CSIT: Suboptimum Transmission Strategy

By using Lemma 2, the conditional channel capacity can be expressed as follows:

$$C_{V_1 H}(P_T;H) = \max_{\substack{P_{T,1}\geq 0,\ P_{T,2}\geq 0\\ P_{T,1}+P_{T,2}\leq P_T}} \{C(P_{T,1};\Sigma_1) + C(P_{T,2};H_2)\} \quad (8)$$

where $P_{T,1}$ is the transmit power allocated to the Σ_1 channel of equation (4), and $P_{T,2}$ is the transmit power on the H_2 channel of equation (5). $C(P_{T,1};\Sigma_1)$ is the conditional channel capacity of the Σ_1 channel with transmit power $P_{T,1}$. Because the Σ_1 channel consists of n parallel Gaussian channels, for a given $P_{T,1}$, the capacity and the optimum power allocation can be obtained similarly to the complete CSIT case [2]. That is,

$$\begin{aligned} C(P_{T,1};\Sigma_1) &= \max_{p(s_1):\ E\{s_1^\dagger s_1\}\leq P_{T,1}} I(s_1;\tilde{y}_1)\\ &= \max_{\substack{P_1\geq 0,\ldots,P_n\geq 0\\ P_1+\cdots+P_n\leq P_{T,1}}} \sum_{i=1}^{n}\log(1+P_i\lambda_i) \quad (9) \end{aligned}$$

where P_i is the power allocated to the i-th transmit symbol and $\lambda_i=\sigma_i^2$ is the i-th largest eigenvelue of $H^\dagger H$ (or $HH^\dagger$).

The second term $C(P_{T,2};H_2)$ in equation (8) is the conditional channel capacity of the H_2 channel with transmit power $P_{T,2}$. In this Section, we confine our attention to a practically reasonable transmission strategy: an equal power allocation for the H_2 channel. Compared to the optimum scheme in [1], the power allocation results in $n+1$ real values in $[0,1]$; therefore, the amount of channel feedback information has been reduced, which is one of advantages of this transmission strategy. The analysis of this transmission scenario is also meaningful because it explains the limiting performance of the systems that comprises of n parallel channels (the Σ_1 channel) from beamforming, for which conventional time-domain only codes would be used; and a MIMO channel (the H_2 channel), for which a space-time code would be employed. In other words, this section assumes that the transmitter has no information about the H_2 channel except the total transmit power $P_{T,2}$ for the channel. Then, as in [2], the conditional capacity expression is given by

$$\begin{aligned} C(P_{T,2};H_2) &= \max_{p(s_2):\ E\{s_2^\dagger s_2\}\leq P_{T,2}} I(s_2;\tilde{y}_2)\\ &= \sum_{i=n+1}^{m}\log\left(1+\frac{P_{T,2}}{t-n}\lambda_i\right) \quad (10) \end{aligned}$$

Combining (9) and (10) with (8), the conditional capacity for a given channel realization H is obtained by solving the following maximization problem:

$$C_{V_1 H}(P_T;H) = \max_{\substack{P_1\geq 0,\ldots,P_n\geq 0,P_{T,2}\geq 0\\ P_1+\cdots+P_n+P_{T,2}\leq P_T}} \Psi(P_1,\ldots,P_n,P_{T,2}), \quad (11)$$

$$\Psi(P_1,\ldots,P_n,P_{T,2}) = \sum_{i=1}^{n}\log(1+P_i\lambda_i) + \sum_{i=n+1}^{m}\log\left(1+\frac{P_{T,2}}{t-n}\lambda_i\right) \quad (12)$$

We can write the constraint maximization using Lagrange multipliers as the maximization of

$$J = \Psi(P_1,\ldots,P_n,P_{T,2}) - \mu\log(e)\cdot\left(\sum_{i=1}^{n}P_i + P_{T,2} - P_T\right)$$

where $-\mu\log(e)$ is the Lagrange multiplier (a constant $-\log(e)$ is included here for a simplicity in the following derivations). Differentiate J with respect to P_i $(1\leq i\leq n)$ and $P_{T,2}$, and set the derivatives to zeros; that is, $\frac{\partial J}{\partial P_i} = 0,\ 1\leq i\leq n,$ and $\frac{\partial J}{\partial P_{T,2}} = 0$. Then, we obtain the following equations:

$$f_i(P_i) \triangleq \frac{\lambda_i}{1+P_i\lambda_i} = \mu, \quad 1\leq i\leq n \quad (13)$$

$$g(P_{T,2}) \triangleq \sum_{i=n+1}^{m}\frac{\lambda_i}{t-n+P_{T,2}\lambda_i} = \mu \quad (14)$$

The power constraint $(\sum_{i=1}^{n}P_i) + P_{T,2} = P_T$ can be rewritten using inverse functions [2] of $f_i(P_i), 1\leq i\leq n$, and $g(P_{T,2})$.

$$\sum_{i=1}^{n}\left[f_i^{-1}(\mu)\right]^+ + \left[g^{-1}(\mu)\right]^+ = P_T \quad (15)$$

Here note that the channel capacity is achieved when the total transmit power equals to P_T. Note also that the inverse function for $f_i(\cdot)$ is easily written by $f_i^{-1}(\mu) = 1/\mu - 1/\lambda_i$, while it is not easy to find a simple expression for $g^{-1}(\cdot)$. The following Theorem summarizes the steps to obtain the conditional channel capacity $C_{V_1 H}(P_T;H)$.

Theorem 1: For a given channel, the channel capacity of MIMO channel with partial CSIT can be obtained by solving for μ satisfying

$$f_i(P_i)=\mu, 1\leq i\leq n;\ g(P_{T,2})=\mu;\ \text{and}$$
$$\sum_{i=1}^{n}\left[f_i^{-1}(\mu)\right]^+ + \left[g^{-1}(\mu)\right]^+ = P_T.$$

where functions $f_i(\cdot)$ and $g(\cdot)$ are defined in (13) and (14). Once the solution μ^* is obtained, the optimum power allocation is given by

$$P_i^* = \left[\frac{1}{\mu^*}-\frac{1}{\lambda_i}\right]^+, 1\leq i\leq n;\ \text{and}\ P_{T,2}^* = \left[g^{-1}(\mu^*)\right]^+ \quad (16)$$

[2]For the existence of inverse functions, we limit the domains of functions $f_i(x)$ and $g(x)$ such that $f_i : (-1/\lambda_i,\infty)\rightarrow(0,\infty)$ and $g : (-(t-n)/\lambda_{n+1},\infty)\rightarrow(0,\infty)$.

And, the conditional channel capacity is given by

$$C_{V_1H}(P_T;H) = \Psi(P_1^*,\ldots,P_n^*,P_{T,2}^*). \qquad (17)$$

where $\Psi(\cdot)$ is defined in (12).

Now, we need to solve for μ that simultaneously satisfies equations (13), (14) and (15). Note that the function $f_i(P_i)$ is a monotonically decreasing function with $f_i(0) = \lambda_i$ and goes to zero as P_i increases; and, so is the function $g(P_{T,2})$ with $g(0) = \frac{1}{t-n}\sum_{i=n+1}^{m}\lambda_i$. A desirable fact is that

$$g(0) = \frac{1}{t-n}\sum_{i=n+1}^{m}\lambda_i \le \frac{1}{m-n}\sum_{i=n+1}^{m}\lambda_i. \qquad (18)$$

Hence, $g(0) \le \lambda_j$, for all $1 \le j \le n$. We now define some parameters to be used in the following discussion:

$$\rho(\mu) = \sum_{i=1}^{n}\left[f_i^{-1}(\mu)\right]^+ + \left[g^{-1}(\mu)\right]^+;\ \text{and}\ \rho_g = \rho(g(0)). \qquad (19)$$

Then, we can solve for μ by considering two cases: i) when $P_T < \rho_g$, and ii) when $P_T \ge \rho_g$. When $P_T < \rho_g$, μ should be greater than $g(0)$; therefore, in equation (15), $\left[g^{-1}(\mu)\right]^+ = 0$. It means that $P_{T,2}$ should be zero, i.e., the H_2 should not be used. Then, the solution μ^* satisfying the three equations (13) – (15) and the channel capacity can be obtained by using normal water-filling just as in the complete CSIT. The optimum power allocation is given by

$$P_i = \left[\frac{1}{\mu^*} - \frac{1}{\lambda_i}\right]^+, 1 \le i \le n;\ \text{and}\ P_{T,2} = 0. \qquad (20)$$

And, the conditional channel capacity is given by

$$C_{V_1H}(P_T;H) = \sum_{i=1}^{n}\left[\log\left(\frac{\lambda_i}{\mu^*}\right)\right]^+. \qquad (21)$$

In the second case when $P_T \ge \rho_g$, μ should be less than $g(0)$; therefore, $P_{T,2}$ is now positive. That is, $P_i = f_i^{-1}(\mu) > 0$ for $1 \le i \le n$, and also $P_{T,2} = \left[g^{-1}(\mu)\right]^+ = g^{-1}(\mu) \ge 0$. The H_2 channel is now being used. Therefore, from equations (13) – (15), we need to solve for μ satisfying

$$\sum_{i=1}^{m}\left(\frac{1}{\mu} - \frac{1}{\lambda_i}\right) + g^{-1}(\mu) = P_T$$

which is equivalent to

$$h(\mu) \triangleq g\left(P_T - \sum_{i=1}^{m}\left(\frac{1}{\mu} - \frac{1}{\lambda_i}\right)\right) - \mu = 0 \qquad (22)$$

The solution μ^* satisfying (22) can be solved numerically by using a zero-finding algorithm for single-variable nonlinear functions. The following Lemma shows the range of μ^* which is helpful in setting up the zero-finding algorithm.

Lemma 3: $\mu^* \in (\mu_L, g(0)]$, and μ_L is given by

$$\mu_L = n\left[P_T + \sum_{i=1}^{n}\frac{1}{\lambda_i} + \frac{t-n}{\lambda_{n+1}}\right]^{-1} \qquad (23)$$

Proof: See Appendix of [6]. ■

Equivalent Water-Filling

From the above derivation of the optimum transmit power allocation, we can see that a MIMO channel with partial CSI at the transmitter has some characteristics of water-filling. In particular, the H_2 channel starts to be used when the transmit power P_T is greater than a certain threshold ρ_g and the power allocation on the Σ_1 channel is determined by the conventional water-filling method. By the following Theorem, we show that the power allocation on the H_2 channel also can understood with an equivalent water-filling model.

Theorem 2: The optimum power allocation over each channel can be viewed as the area determined by the the following function that defines the shape of the vessel for water-filling.

$$v(y) = \begin{cases} 0 & \text{if } 0 \le y < 1/\lambda_1, \\ 1 & \text{if } 1/\lambda_1 \le y < 1/\lambda_2, \\ \vdots & \quad\vdots \\ n-1 & \text{if } 1/\lambda_{n-1} \le y < 1/\lambda_n, \\ n & \text{if } 1/\lambda_n \le y < 1/g(0), \\ f(y) & \text{if } y \ge g(0). \end{cases} \qquad (24)$$

where $f(y)$ is given by

$$f(y) = \frac{1}{y^2}\left[\sum_{i=n+1}^{m}\frac{\lambda_i^2}{(t-n+g^{-1}(1/y)\lambda_i)^2}\right]^{-1} + n \qquad (25)$$

Then, the optimum power allocation can be written as follows.

$$P_i = \int_{i-1}^{i}\left[\nu - \frac{1}{\lambda_i}\right]^+ dx = \left[\nu - \frac{1}{\lambda_i}\right]^+,\ \text{for } 1 \le i \le n$$

$$P_{T,2} = \int_{1/g(0)}^{\nu}[v(y) - n]dy\ \text{ if } \nu \ge 1/g(0);\ = 0\ \text{ otherwise}$$

where $\nu = 1/\mu$ is the level of water-filling.

Proof: See Appendix of [6]. ■

Figure 1 shows an example of the equivalent water-filling shape that was calculated numerically from Theorem 2. The shape of the equivalent water-filling explains water-filling characteristics. Since, for $1 \le i \le n$, the width of the i-th channel is one, function $f_i^{-1}(\nu)$ has unit slope. And, the last H_2 channel is a nonlinear function which results in $P_{T,2}(\nu) < (m-n)\left(\nu - \frac{1}{g(0)}\right)$.

If we approximate the water-filling vessel to the rectangular one depicted in Figure 1, then the calculation for the power allocation, therefore, also the channel capacity, will become much easier. It can be shown that a lower bound for the capacity is achieved with the simple rectangular approximation (refer to [6] for details).

V. Numerical Results

Although the proposed system is irrelevant to the channel model, for numerical comparisons, we considered the MIMO channel that was assumed in [2]. The channel gain matrix $H \in \mathbb{C}^{r\times t}$ is a random matrix independent to the transmit symbols s and the additive noise η, with *i.i.d.* entries, each having independent real and imaginary parts with zero-mean and variance 1/2.

Figure 2 is ergodic capacities versus total transmit power with different CSI assumptions and transmission strategies for MIMO channel with parameters $t = 4$ and $r = 2$. In order to effectively compare the performance of different transmission strategies, each capacity has been normalized to $C_{HH}(P_T)$, the capacity of *complete CSIT*. $C_{\phi H}$ is the capacity of *no CSIT*, i.e., $n = 0$ and equal power allocation. $C_{V_1 H}^{(\text{opt})}$ and $C_{\phi H}^{(\text{opt})}$ are the capacities of the optimum transmission strategy in [1], with V_1 and nothing ($n = 0$) as spatial information at the transmitter, respectively. In comparing $C_{\phi H}^{(\text{opt})}$ and $C_{V_1 H}(n = 1)$, it is noticeable that at low transmit power $C_{\phi H}^{(\text{opt})} < C_{V_1 H}(n = 1)$, but at intermediate and high transmit power $C_{\phi H}^{(\text{opt})}$ is superior. This observation implies that when the transmit power is low the spatial information of the channel is important, and as the transmit power increases the power allocation is becoming meaningful from a capacity point of view. Note that the channel feedback information required for the two strategies are different: for first strategy, t real values in $[0, 1]$, i.e., $(\gamma_1, \ldots, \gamma_t)$; and for the second one is one t-dimensional complex vector v_1 and one real value γ_1 in $[0, 1]$ (γ_2 is determined from γ_1 as $1 - \gamma_1$).

Generally speaking, in spatially correlated MIMO channels, the gains of spatial channels are more separated than in *i.i.d.* channels. Therefore, the proposed scheme is expected to be more beneficial in spatially correlated channels. This was verified by simulation using the channel model of [7] and [8] (the results are not shown in this paper due to limited space).

VI. Conclusion

We considered multiple antenna systems consisting of t transmit and r receive antennas, and partial channel state information available at the transmitter. A transmission method was considered that decomposes the MIMO channel into two parts: n parallel channels and a new small coupled MIMO channel. This paper derived the channel capacity of the multiple antenna channels employing a reasonable coding strategy in which conventional time-domain only code is used for each of the parallel channels and a space-time code for the small MIMO channel. An equivalent water-filling model for the proposed MIMO channel was also derived. The simulation results have shown that, with this suboptimum strategy, performance similar to the optimum strategy can be achieved with reduced complexity in computing the power allocation.

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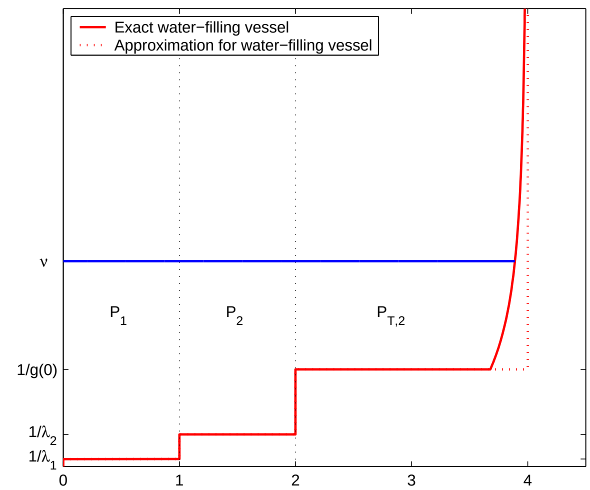

Fig. 1. Equivalent water-filling by Theorem 2 ($t = 4, r = 4, n = 2, \lambda_1 = 9.6303, \lambda_2 = 2.2467, \lambda_3 = 1.0682, \lambda_4 = 0.4174$).

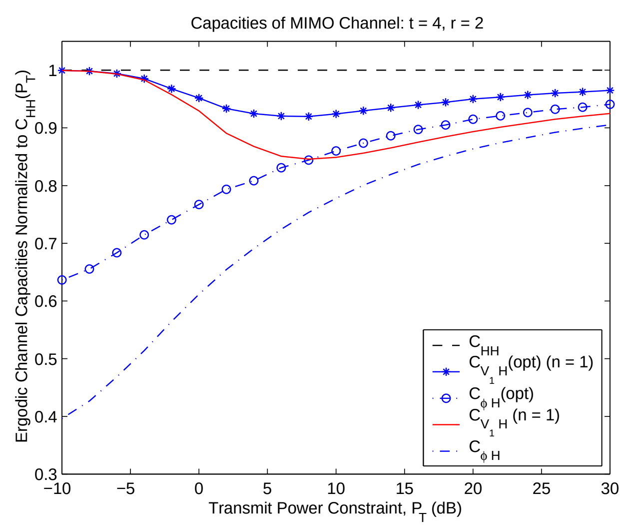

Fig. 2. Ergodic capacities of MIMO channel with different CSI assumptions and transmission strategies ($t = 4$ and $r = 2$).