Adaptive Modulation for Multiple Antenna Channels

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***Abstract*— We consider the use of adaptive modulation scheme for multiple transmit and multiple receive antenna system with instantaneous channel information known to both the receiver and the transmitter. We derive an efficient bit allocation algorithm which maximizes the transmission rate for a given transmit power. The algorithm is generally a greedy algorithm; however, we derive a sufficient condition for the bit allocation algorithm to be globally optimal, which is found to be satisfied in all digital modulation schemes. It is found that when uncoded M-ary QAM is used with a target symbol error probability of 10^{-5} there is about 9 dB gap between the channel capacity and the throughput of adaptive modulation.**

I. Introduction

The adaptive modulation for scalar channels was studied in [1]. The fundamental concept of adaptive modulation is that the system parameters in the physical layer are adaptively changed based on the channel status to increase the communication link quality, mostly, transmission rate. This paper considers using the adaptive modulation method in multiple antenna channels. In particular, we focus on the power allocation problem over multiple spatial channels.

In recent years, multiple antenna communication systems have gathered much attention for high-rate transmission over wireless channels. Telatar [2] showed the information-theoretic capacity of multiple-input multiple-output (MIMO) channels with flat fading.

If the channel state information is known to both the transmitter and the receiver, an MIMO channel can be decomposed into parallel independent single-input single-output (SISO) channels by employing appropriate operations at the transmitter and the receiver. The resulting decomposed channels are characterized by the channel gain matrix, i.e., the gains of the decomposed channels are determined to be the singular values of the channel gain matrix. After decomposing the channel into parallel channels, the remaining problem is how to allocate the transmit power over the decomposed channels to maximize the total transmission rate. The adaptive modulation for multiple antenna channels is concerned with adaptation of modulation parameters in spatial as well as temporal domain.

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The power allocation problem can be equivalently considered as so-called *bit allocation* problem[1] over multiple spatial channels if the target link quality is fixed. The bit allocation problem involves solving an optimization problem with integer variables, in which the optimum bit allocation over the multiple channels is determined to minimize the total transmit power for a given number of transmission bits. That is, the cost function is the total required transmit power for transmitting the given number of bits with a target link quality satisfied. The channel gain matrix characterizes the cost function.

One contribution of this paper is that the bit allocation problem is formulated as an optimization problem, and then an efficient bit allocation algorithm is derived. The derived algorithm is a greedy algorithm, which generally may not be the global optimum. However, we derive a sufficient condition for the bit allocation algorithm to be globally optimal, which is found to be satisfied in all M-ary digital modulation schemes.

We are also interested in the average transmission rate that can be affordable with adaptive modulation in MIMO systems and how far the average rate is away from the channel capacity. We consider an adaptive modulation scheme that changes the modulation order of M-ary QAM, $M \in \{0, 2, 2^2, \ldots, 2^8\}$, depending on the channel state. The simulation results show that the average transmission rate of the adaptive uncoded M-ary QAM is about 9 dB away from the channel capacity when the target symbol error probability is set to 10^{-5}.

II. System Model

We consider a point-to-point flat fading channel with multiple antennas at both the transmitter and the receiver. The number of transmit antennas is denoted by t and the number of receive antennas by r. We consider a linear discrete channel model

$$\mathbf{y} = H\mathbf{x} + \mathbf{w} \tag{1}$$

where $\mathbf{x} \in \mathbb{C}^{t\times 1}$ is the transmitted signal, $\mathbf{y} \in \mathbb{C}^{r\times 1}$ is the received signal, $H \in \mathbb{C}^{r\times t}$ is the channel gain matrix, and $\mathbf{w} \in \mathbb{C}^{r\times 1}$ is the zero-mean complex Gaussian noise with covariance $E\{\mathbf{w}\mathbf{w}^\dagger\} = I_r$. As in [2], we assume that H is a random matrix independent of $\mathbf{x}$ and $\mathbf{w}$. H is a complex Gaussian

[1] The bit allocation problem has been also studied for multi-carrier communication applications; there, computationally efficient suboptimum schemes have been focused because the number of parallel channels is usually large.

matrix with *i.i.d.* entries, each entry having independent real and imaginary parts with zero-mean and variance 1/2. And, we assume a power constraint on the transmitted signal with $E\{\mathbf{x}^\dagger\mathbf{x}\} \le P_T$.

A. Channel Decomposition Using SVD

If the channel state information is known at both the transmitter and the receiver, the channel can be decomposed into parallel non-interfering SISO channels by using singular value decomposition (SVD): $H = U\Sigma V^\dagger$, where $U \in \mathbb{C}^{r\times r}$ and $V \in \mathbb{C}^{t\times t}$ are unitary, and $\Sigma \in \mathbb{R}^{r\times t}$ contains the sigular values with σ_i is the i-th singular value of H. If the transmitter knows the channel matrix H (or V), $\mathbf{x} = V\mathbf{s}$ is transmitted where $\mathbf{s} \in \mathbb{C}^{t\times 1}$ is the modulation symbol vector. And, at the receiver, by pre-multiplying $\mathbf{y}$ by $U^\dagger$ we have

$$\tilde{\mathbf{y}} = \Sigma\mathbf{s} + \tilde{\mathbf{w}} \tag{2}$$

where $\tilde{\mathbf{y}} = U^\dagger\mathbf{y}$, and $\tilde{\mathbf{w}} = U^\dagger\mathbf{w}$. Since $m \triangleq \text{rank}(H) \le \min(t, r)$, (2) can be rewritten as

$$\tilde{y}_i = \begin{cases} \sigma_i s_i + \tilde{w}_i, & 1 \le i \le m \\ \tilde{w}_i, & \text{otherwise} \end{cases} \tag{3}$$

where the subscript i indicates the i-th element of the corresponding vector. Note that since U and V are unitary matrix, $E\{\mathbf{s}^\dagger\mathbf{s}\} = E\{\mathbf{x}^\dagger\mathbf{x}\}$ and $E\{\tilde{\mathbf{w}}\tilde{\mathbf{w}}^\dagger\} = E\{\mathbf{w}\mathbf{w}^\dagger\}$. From (3), we can see that, for a given channel H, we have m independent parallel Gaussian channels with the i-th channel having a gain σ_i. Therefore, the demodulation for the transmitted symbol vector $\mathbf{s}$ becomes simple: just demodulate each channel independently with a decision variable $\tilde{y}_i$.

B. Transmit Beamforming and Maximal-Ratio Combining

We also consider a transmit beamforming and maximal-ratio combining (MRC) at the receiver for comparison. In this case, the modulation symbol is now a scalar s and the principle eigenvector of $H^\dagger H$ is employed as the beamforming weight, i.e., $\mathbf{x} = \mathbf{v}_1 s$. And, at the receiver side, the weighting vector is set to $\mathbf{h} = H\mathbf{v}_1/\|H\mathbf{v}_1\|$, which equals to $\mathbf{u}_1$, the principle eigenvector of $HH^\dagger$, then the decision variable is given by

$$\tilde{y} = \mathbf{h}^\dagger\mathbf{y} = \sigma_1 s + \tilde{w}. \tag{4}$$

The resulting channel is a single Gaussian channel with a channel gain σ_1 and $E\{|\tilde{w}|^2\} = 1$. We place the same input power constraint, $E\{|s|^2\} \le P_T$.

C. M-ary QAM

We consider M-ary QAM with $M \in \{0, 2^1, 2^2, \ldots, 2^8\}$, which corresponds to transmitting $\{0, 1, 2, \ldots, 8\}$ bits per channel use. Since the decision boundary in the QAM constellation is not rectangular when $M = 32$ and 128 as shown in Figure 1, we use Craig's method [3] to obtain the exact numerical values for the probability of symbol error. Table I summarizes the required symbol energy per noise density, which is denoted by $g(n)$ for $M = 2^n$, when the target symbol error probability (SER) is 10^{-5}.

TABLE I

THE REQUIRED SYMBOL ENERGY PER NOISE DENSITY IN dB, $g(n)$, OF M-ARY QAM WHEN THE TARGET SYMBOL ERROR PROBABILITY IS 10^{-5}.

M	0	2	4	8	16	32	64	128	256
n	0	1	2	3	4	5	6	7	8
$g(n)$	–	9.6	12.9	17.8	20.1	23.1	26.4	29.3	32.5

III. Optimum Bit Allocation

When the channel state information is known to both the transmitter and the receiver, after decomposing the channel into m parallel channels as in the previous section, the remaining problem is how to allocate the transmit power over the decomposed channels to maximize the total transmission rate. One obvious strategy from information theory is to allocate more power to better channel as in *water-filling* [4]. The water-filling assumes that there exists infinitely many *continuous* levels of power. But, in the adaptive modulation with M-ary modulation, we need *discrete-valued* power to transmit one more additional bit with a given link quality satisfied. Therefore, the power allocation in adaptive modulation for MIMO systems is to find the optimum way to allocate the available transmit power over multiple parallel channels with each having its *discrete* levels which are determined by the modulation scheme and its channel gain.

Since the channel gain of the i-th decomposed channel is σ_i, the received signal-to-noise ratio (SNR) is given by $\sigma_i^2 P_i$, where P_i is the *transmit* SNR of the i-th channel. Note that since we assume a normalized noise with unit variance, the transmit power on a decomposed channel equals to the transmit SNR. Therefore, if we denote the required SNR for $M = 2^n$, i.e. n bits transmission per channel use, by $g(n)$, the required transmit power with a channel gain σ_i is given by

$$\Gamma_i(n) = \frac{g(n)}{\sigma_i^2}, \;\; n = 1, \ldots, n_{\max}. \tag{5}$$

Consider that we transmit $n_i \in \mathbb{N} \triangleq \{0, 1, 2, \ldots, n_{\max}\}$ bits over the i-th channel, $1 \le i \le m$, we call this a *bit allocation* $\mathbf{n} = (n_1, \ldots, n_m) \in \mathbb{N}^m$. Then, the total transmit power, the cost function in this optimization problem, is given by

$$f(\mathbf{n}) = f(n_1, \ldots, n_m) = \sum_{i=1}^{m} \Gamma_i(n_i) = \sum_{i=1}^{m} a_i g(n_i) \tag{6}$$

where $a_i = 1/\sigma_i^2$ and without loss of generality we assume that $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_m > 0$, so $0 < a_1 \le a_2 \le \ldots \le a_m$.

For simpler notation, we define

$$\Delta_n \triangleq g(n) - g(n-1),$$

and $\mathbf{n}_{(i)-1}$ as a vector with same elements of $\mathbf{n}$ except i-th element reduced by one. Since we are dealing with a non-negative integer $n_i \in \mathbb{N}$, it is more useful to define

$$\mathbf{n}_{(i)-1} \triangleq \left(n_1, \ldots, n_{i-1}, [n_i - 1]^+, n_{i+1}, \ldots, n_m\right). \tag{7}$$

where $[x]^+ \triangleq \max\{0, x\}$.

Theorem 1: Consider two bit allocations, $\mathbf{n}$ and $\mathbf{k}$, with $n_i \geq k_i$ for some i and the total transmit powers with two bit allocations satisfying $f(\mathbf{n}) \leq f(\mathbf{k})$. Then,

$$f(\mathbf{n}_{(i)-1}) \leq f(\mathbf{k}_{(i)-1}) \tag{8}$$

with a sufficient condition

$$0 < \Delta_{k_i} \leq \Delta_{n_i}. \tag{9}$$

Proof: Since $f(\mathbf{n}) - f(\mathbf{k}) \leq 0$, the hypothesis on the total transmit powers can be rewritten as

$$\sum_{j=1}^{m} a_j [g(n_j) - g(k_j)] \leq 0$$

After subtracting the i-th term from both sides, we have

$$\sum_{\substack{j=1 \\ j \neq i}}^{m} a_j [g(n_j) - g(k_j)] \leq a_i [g(k_i) - g(n_i)]$$

From the above, we can show the inequality (8).

$$\begin{aligned}
& f(\mathbf{n}_{(i)-1}) - f(\mathbf{k}_{(i)-1}) \\
& = a_i [g(n_i - 1) - g(k_i - 1)] + \sum_{\substack{j=1 \\ j \neq i}}^{m} a_j [g(n_j) - g(k_j)] \\
& \leq a_i [g(n_i - 1) - g(k_i - 1)] + a_i [g(k_i) - g(n_i)] \\
& = a_i \{[g(k_i) - g(k_i - 1)] - [g(n_i) - g(n_i - 1)]\} \\
& = a_i (\Delta_{k_i} - \Delta_{n_i}) \leq 0
\end{aligned}$$

The last inequality is satisfied under the sufficient condition (9). Hence, the theorem was proved. ■

Example 1: Assume $m = 2$, $\mathbb{N} = \{0, 1, \ldots, 8\}$, and bit allocation $\mathbf{n} = (7, 5)$ is the optimal in transmitting 12 bits per channel user (see Figure 2). That is,

$$f(7, 5) = \min_{\substack{k_1, k_2 \in \mathbb{N} \\ k_1 + k_2 = 12}} f(k_1, k_2).$$

Applying Theorem 1,

$$f(6, 5) \leq f(5, 6), f(4, 7), f(3, 8) \text{ and } f(7, 4) \leq f(8, 3)$$

Therefore,

$$\min_{\substack{k_1, k_2 \in \mathbb{N} \\ k_1 + k_2 = 11}} f(k_1, k_2) = \min\{f(6, 5), f(7, 4)\}.$$

We want to generalize the idea in the above Example and derive an efficient bit allocation algorithm based on Theorem 1. Before doing that, we need a set of definitions for the subsets of all possible bit allocations, $\mathbb{N}^m$.

$$\mathcal{S}^N \triangleq \{\mathbf{n} : \sum_{i=1}^{m} n_i = N \text{ and } n_i \in \mathbb{N}, i = 1, \ldots, m\}$$

which is the set of all bit allocations that are corresponding to N bits transmission per channel use. We also define a subset of $\mathcal{S}^N$ with a parameter $\mathbf{a} \in \mathcal{S}^N$ as set of all the vectors in $\mathcal{S}^N$ with the i-th element less than the i-th element of $\mathbf{a}$, i.e.,

$$\mathcal{D}_i^N(\mathbf{a}) \triangleq \left\{\mathbf{n} : \mathbf{n} \in \mathcal{S}^N \text{and } n_i \leq a_i\right\}, i = 1, \ldots, m.$$

We can see that $\mathcal{S}^N = \bigcup_{i=1}^m \mathcal{D}_i^N(\mathbf{a}),$ for some $\mathbf{a} \in \mathcal{S}^N$. And, we also define

$$\mathcal{D}_{(i)-1}^N(\mathbf{a}) \triangleq \left\{\mathbf{n}_{(i)-1} : \mathbf{n} \in \mathcal{S}_i^N(\mathbf{a})\right\}.$$

Then, from the above definitions we can easily see the following Lemma.

Lemma 1: For some vector $\mathbf{a} \in \mathcal{S}^N$,

$$\mathcal{S}^{N-1} = \bigcup_{i=1}^{m} \mathcal{D}_{(i)-1}^N(\mathbf{a}). \tag{10}$$

We can divide the minimizing problem over a variable space, say $\mathcal{S}$, into two stages: First, do m number of minimizations over the subset variable spaces, say $\mathcal{D}_i$, $i = 1, \ldots, m$, where $\mathcal{S} = \bigcup_{i=1}^m \mathcal{D}_i$; and secondly, find the final minimizing point among the m minimizing points obtained from the first step. The following Lemma summarizes this idea.

Lemma 2: Suppose the variable space for the optimization, $\mathcal{S} = \bigcup_{i=1}^m \mathcal{D}_i$, and $\mathbf{n}_i^*$ is the minimizing point over the subset space $\mathcal{D}_i$, i.e., $f(\mathbf{n}_i^*) = \min_{\mathbf{n} \in \mathcal{D}_i} f(\mathbf{n})$. Then, the minimizing point $\mathbf{n}^*$ over the whole space $\mathcal{S}$ is given by

$$f(\mathbf{n}^*) = \min_{\mathbf{n} \in \mathcal{S}} f(\mathbf{n}) = \min_i f(\mathbf{n}_i^*). \tag{11}$$

Lemma 3: Suppose that $\mathbf{a}$ is the minimizing point in $\mathcal{S}^N$, i.e., $f(\mathbf{a}) = \min_{\mathbf{n} \in \mathcal{S}^N} f(\mathbf{n})$ and that $g(n)$ satisfies the following condition,

$$0 < \Delta_1 \leq \Delta_2 \leq \ldots \leq \Delta_{n_{\max}}. \tag{12}$$

Then, $\mathbf{a}_{(i)-1}$ is the the minimizing point in $\mathcal{D}_{(i)-1}^N(\mathbf{a}) \subset \mathcal{S}^{N-1}$. That is,

$$f(\mathbf{a}_{(i)-1}) = \min_{\mathbf{n} \in \mathcal{D}_{(i)-1}^N(\mathbf{a})} f(\mathbf{n}) \quad \text{for } i = 1, \ldots, m. \tag{13}$$

Proof: Since $f(\mathbf{a}) = \min_{\mathbf{n} \in \mathcal{S}^N} f(\mathbf{n})$, $f(\mathbf{a}) \leq f(\mathbf{k})$ for all $\mathbf{k} \in \mathcal{S}^N$. By Theorem 1, it implies that

$$f(\mathbf{a}_{(i)-1}) \leq f(\mathbf{k}_{(i)-1}) \text{ for all } \mathbf{k} \in \mathcal{S}^N. \tag{14}$$

Since we defined $\mathcal{D}_{(i)-1}^N(\mathbf{a}) = \left\{\mathbf{k}_{(i)-1} : \mathbf{k} \in \mathcal{S}_i^N(\mathbf{a})\right\}$, (14) is equivalent to (13). ■

From Lemma 1, 2 and 3, we can derive the following Theorem which is directly related to the optimum bit allocation algorithm.

Theorem 2: If $\mathbf{a}$ is the bit allocation with N bits transmission that minimizes the transmit power, i.e., $f(\mathbf{a}) = \min_{\mathbf{n} \in \mathcal{S}^N} f(\mathbf{n})$, then the optimum bit allocation $\mathbf{b}$ with $N - 1$ bits transmission is given by

$$f(\mathbf{b}) \triangleq \min_{\mathbf{n} \in \mathcal{S}^{N-1}} f(\mathbf{n}) = \min_i f(\mathbf{a}_{(i)-1}). \tag{15}$$

Proof: For the optimization problem over $\mathcal{S}^{N-1}$, we consider the subsets $\mathcal{D}^N_{(i)-1} \subset \mathcal{S}^{N-1}, 1 \le i \le m$, as in Lemma 1, and apply two steps optimization stated in Lemma 2, then we can easily arrive the result by Lemma (3). ■

From the above Theorem 2, we derive an efficient bit allocation algorithm as is summarized as follows:

1) Start with $N = m \cdot n_{\max}$; and $\mathbf{n}^* = (n_{\max}, \ldots, n_{\max})$.
2) $N \leftarrow N - 1$; $i^* = \arg\min_{1\le i\le m} f(\mathbf{n}^*_{(i)-1})$, then $\mathbf{n}^* \leftarrow \mathbf{n}^*_{(i^*)-1}$.
3) Repeat Step 2 until $f(\mathbf{n}^*) \le P_T$. Otherwise, stop.

where $\mathbf{n}^*_{(i^*)-1}$ as a vector with same elements of $\mathbf{n}^*$ except i^*-th element reduced by one as defined in (7).

Since the cost function, the total transmit power, has a form of $f(\mathbf{n}) = \sum_{i=1}^m a_i g(n_i)$, by noticing the fact $f(\mathbf{n}^*_{(i)-1}) = f(\mathbf{n}^*) - a_i \Delta_{n_i^*}$, the second step in the above bit allocation algorithm is equivalent to

2′) $N \leftarrow N - 1$; $i^* = \arg\max_{1\le i\le m} a_i \Delta_{n_i^*}$, then $\mathbf{n}^* \leftarrow \mathbf{n}^*_{(i^*)-1}, f(\mathbf{n}^*) \leftarrow f(\mathbf{n}^*) - \max_i a_i \Delta_{n_i^*}$.

The above bit algorithm always gives the bit allocation that maximizes the total number of transmission bits for a given power constraint, since the resulting bit allocation $\mathbf{n}^*$ for each N always provides the minimum transmit power. The algorithm is, in general, a *greedy algorithm* which may not be the global optimum, since it follows the best way at each stage without care of the previous path, regardless of the characteristics of the cost function. In this section, we have shown that the derived bit allocation algorithm provides the globally optimum point under a sufficient condition (12), which is satisfied for all M-ary digital modulation schemes.

One can also develop, with the same principle, an equivalent bit allocation algorithm that goes the reverse direction:

1) Start with $N = 0$; and $\mathbf{n}^* = (0, \ldots, 0)$.
2) $N \leftarrow N + 1$; $\mathbf{n}_{\text{tmp}} \leftarrow \mathbf{n}^*$; $i^* = \arg\min_{1\le i\le m} a_i \Delta_{n_i^*+1}$, then $\mathbf{n}^* \leftarrow \mathbf{n}^*_{(i^*)+1}, f(\mathbf{n}^*) \leftarrow f(\mathbf{n}^*) + \min_i a_i \Delta_{n_i^*+1}$.
3) Repeat Step 2 until $f(\mathbf{n}^*) > P_T$. Otherwise, stop and $\mathbf{n}^* \leftarrow \mathbf{n}_{\text{tmp}}$.

where $\mathbf{n}_{(i)+1} \triangleq (n_1, \ldots, n_{i-1}, n_i + 1, n_{i+1}, \ldots, n_m)$.

IV. Throughput of Adaptive M-ary QAM

The ergodic channel capacity of MIMO channels with perfect channel state information known to both the transmitter and the receiver is given by

$$C(P_T) = E_H\{C(P_T; H)\} \tag{16}$$

where $E_H\{\cdot\}$ is the expectation over the random channel matrix H, and $C(P_T; H)$ is the conditional capacity for a given channel realization H, which is given by [2]

$$C(P_T; H) = \sum_{i=1}^m \left[\log_2(\nu\sigma_i^2)\right]^+ \tag{17}$$

and ν is the water-filling level satisfying

$$\sum_{i=1}^m \left[\nu - 1/\sigma_i^2\right]^+ = P_T. \tag{18}$$

The cost function of (6) is determined by the channel matrix H, or more precisely by $\{\sigma_1, \ldots, \sigma_m\}$; and, for a channel realization, the optimum bit allocation can be solved with the algorithm described in the previous section. Then, the maximum number of transmission bits, let us say $R(P_T; H) = R(\sigma_1, \ldots, \sigma_m; P_T)$, is simply the sum of all the elements of the bit allocation, i.e.,

$$R(P_T; H) = \sum_{i=1}^m n_i^*,$$

where n_i^* is the i-th element of the optimum bit allocation $\mathbf{n}^*$. Therefore, for a given transmit power constraint P_T, the average bits per channel use, that is offered by adaptive modulation, can be expressed as,

$$\begin{aligned} R(P_T) &= E_H\{R(P_T; H)\} \\ &= \int \cdots \int R(\sigma_1, \ldots, \sigma_m; P_T) p(\sigma_1, \ldots, \sigma_m)\, d\sigma_1 \cdots d\sigma_m \end{aligned} \tag{19}$$

where $p(\sigma_1, \ldots, \sigma_m)$ is the joint probability density function of $\{\sigma_i\}$, which can be found in literature, e.g., [2]. However, unfortunately, it is not easy to find any closed-form expression for $R(\sigma_1, \ldots, \sigma_m; P_T)$. Instead, we resort to the solutions from the bit allocation algorithm.

We ran simulations to evaluate the throughput of adaptive modulation. We considered M-ary QAM with required SER of 10^{-5}, and used the required transmit power (or transmit SNR) for each transmission bit, $g(n), n = 0, 1, \ldots, 8$, that are shown in the Table I in Section II. Figures 3 shows the results when $t = \{1, 2, 3, 4\}$ and $r = 2$. We can see that the throughput of adaptive modulation is away 9–10 dB from the channel capacity calculated from (16). One apparent observation is that as P_T increase $R(P_T) \to m\, n_{\max}$, because at high P_T all m channels are utilized with $n_{\max}$ bits transmission on each channel. Before $R(P_T)$ is saturated, the throughput is almost parallel to $C(P_T)$.

One obvious way to reduce the gap to the channel capacity is to employ channel codes. If the perfect channel is known to both sides so that the MIMO channel can be decomposed into non-interfering parallel channels, there is no need to use vector channel coding that needs high complexity, and conventional scalar channel coding for each channel is sufficient. The throughput of adaptive modulation with independent scalar codings can be obtained using a similar approach, because we separated the power allocation problem in the MIMO channel from the modulation issues. That is, once the required transmit power $\{g(n)\}$ that accounts for the effect of channel coding is obtained, we can obtain the throughput by just substituting the new $\{g(n)\}$ in the bit allocation algorithm.

The beamforming strategy described in Section II needs less information about the channel (only $\mathbf{v}_1$ and σ_1 are necessary at the transmitter). In Figure 4, we compared the performance of the transmit beamforming with the channel decomposition method using SVD. When P_T is low, the transmit beamforming scheme shows a comparable performance to the channel decomposition method. This is because at low transmit power only the best spatial channel is used most of the time. Therefore, the two schemes give little difference in performance. But, as P_T increases, the other remaining channels are starting to be utilized; hence, the channel decomposition method outperforms the transmit beamforming counterpart.

V. Conclusion

We consider the use of adaptive modulation scheme for MIMO channels with instantaneous channel information known to both the receiver and transmitter. An efficient bit allocation algorithm is derived and its optimality is proved under a sufficient condition, which is satisfied in all M-ary modulation schemes. It is found that when M-ary QAM is used with a target symbol error probability of 10^{-5} there is about 9 dB gap between the channel capacity and the throughput of adaptive modulation. And, it is also shown that a transmit beamforming scheme, which needs less channel information, gives a comparable performance when the power constraint is low. Relaxing the assumption of perfect channel information at both communication sides needs further studies.

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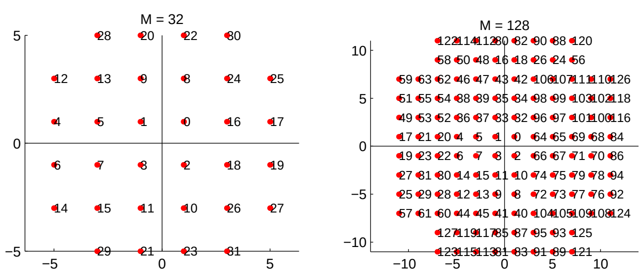

(a) $M = 32$ (b) $M = 128$

Fig. 1. Signal constellation of M-ary QAM ($M = 32, 128$).

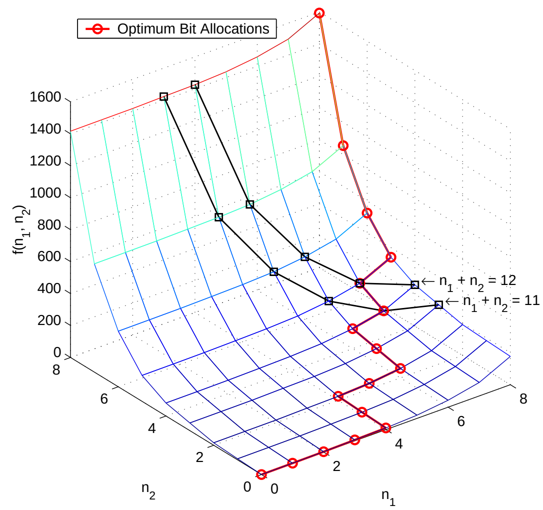

Fig. 2. Example of bit allocation algorithm ($m = 2, n_{\max} = 8$).

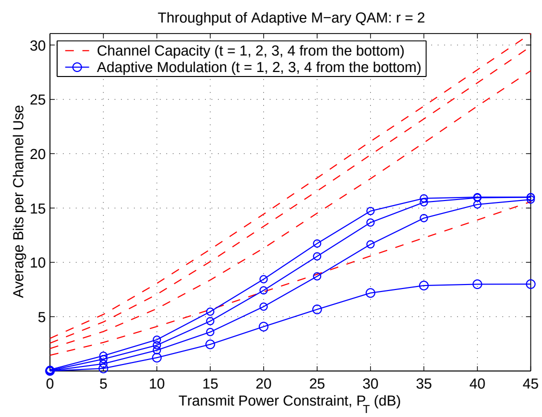

Fig. 3. Average bits per channel use of M-ary QAM adaptive modulation when $r = 2$ fixed. The solid lines are for adaptive modulation with $t = 1, 2, 3, 4$ (from the bottom), and the dashed lines for the channel capacities with $r = 2$ and $t = 1, 2, 3, 4$ (from the bottom).

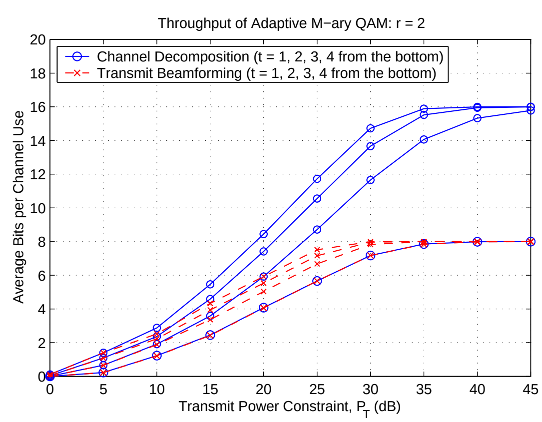

Fig. 4. Average bits per channel use of M-ary QAM adaptive modulation when $r = 2$ fixed. The solid lines are for channel decomposition method, and the dashed lines for the transmit beamforming scheme, with $t = 1, 2, 3, 4$ (from the bottom).