Stochastic Control for Energy Efficient Resource Allocation in Wireless Networks

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Outline

1. Introduction and Scope of the Thesis
2. Power Optimal Scheduling
   - Minimum Rate Guarantee
   - Fairness Guarantee
   - Average Delay Guarantee - point To point Link
     - Learning Algorithms: Overview
     - Problem Formulation
   - Energy Efficient Video Transmission
3. Summary and Future Scope
Motivation

- Wireless LANs, ad-Hoc networks, sensor networks
  limited battery life, bandwidth
- Maintain acceptable QoS metric
  rate, delay, fairness
- Efficient utilization of limited resources
- Time varying wireless channel conditions
  - Exploit channel variations
    - opportunistic scheduling:
      Schedule user with the best channel condition
  - Can we perform power control to exploit channel variations?
  - Joint opportunistic and energy efficient control
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Power Optimal Scheduling

- Minimum rate guarantee (Multi-user)
- Fairness guarantee (Multi-user)
- Average delay guarantee with finite buffer (point to point link)
Shannon’s capacity:

\[ P = \frac{N_0 W}{x} \left( e^{u/W} - 1 \right), \]

- \( N_0 \): Spectral density of AWGN channel
- \( W \): Spectrum bandwidth
- \( u \): Transmission rate

Stochastic approximation:

\[ \lambda(n + 1) = \lambda(n) + \alpha(n) (H(\lambda(n)) + M(n + 1)) \tag{1} \]

\[ E[h(\lambda, x)] = H(\lambda) \]

Martingale \( M(n + 1) = h(\lambda(n), x(n)) - H(\lambda(n)) \)

If step sizes satisfy \( \sum_{n=0}^\infty \alpha(n) = \infty, \quad \sum_{n=0}^\infty \alpha(n)^2 < \infty \)

then (1) tracks the Ordinary Differential equation (ODE),

\[ \dot{\lambda}(t) = H(\lambda(t)) \tag{2} \]
System Model

- Slotted single-hop TDMA system
- Uplink scheduling
- Perfect channel state information
- Channel process ergodic (i.i.d. or Markovian)

Figure: Single hop system model
Problem Formulation

**Minimize average power**

\[
\min \limsup_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} q(n),
\]

**Subject to average rate constraints** \( C_i \)

\[
\limsup_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} U_i(q_i(n), x_i(n))) \geq C_i \quad \forall i,
\]

\[
q(n) \geq 0,
\]

\[
\sum_{i=1}^{N} y_i(n) \leq 1 \quad \forall n \quad (3)
\]

- \( U \) is information theoretic rate and is concave differentiable function of \( x_i, q_i \)

\[
U = \log(1 + x_i q_i)
\]

- \( x = (x_1, x_2, \cdots, x_N) \)
- \( y = (y_1, y_2, \cdots, y_N) \)
Problem Formulation

- **Minimize average power**

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\[ q(n) \geq 0, \]

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- \( y = (y_1, y_2, \cdots, y_N) \)
Multiuser Optimal Solution

Proposition

Optimal Policy for multiple users is to select $k^{th}$ user and transmit with power $q^*$

Sketch of Proof

- Use ergodicity to convert optimization problem (3) in continuous domain
- Minimize Lagrangian of (3) w.r.t. $q$ first, then w.r.t. $y$
- Optimal power for single user,
  $$q_i^* = \left( \lambda_i - \frac{1}{x_i} \right)^+,$$
  where $\lambda_i$ is the Lagrange multiplier
- Minimizing w.r.t. $y$, we get,
  $$k = \arg\min_i (q_i^* - \lambda_i [\log(1 + q_i^* x_i) - C_i])$$
Online Algorithm (1)

- After minimizing over the primal variables, optimal value of Lagrangian is,

$$ F(\lambda) = \left[ E \left( \min_i (q_i^* - \lambda_i \log(1 + q_i^* x_i(n)) - C_i) \right) \right] $$

- $F(\lambda)$ strictly concave $\rightarrow$ unique maximum
- Need to find saddle point

- Consider for example $f(x)$ is continuous differentiable
- Gradient ascent scheme for maximizing $f$ is,

$$ x_{n+1} = x_n + \alpha_n \dot{f}(x_n) $$

- It tracks the differential equation,

$$ \dot{x}(t) = \frac{df}{dx} = \dot{f} $$
Online Algorithm (2)

- Estimate $\lambda_i$ online

$$
\lambda_i(n+1) = \{\lambda_i(n) - \alpha(n) [y_i(n) \log \left(1 + \left(\frac{\lambda_i(n)}{x_i(n)}\right)^+ x_i(n)\right)] - C_i\}^+ \quad \forall i,
$$

**Figure:** Block diagram for on-line policy
Optimality and Stability of Update Equation

- Iterations converge to differential inclusion almost surely to
  \[ \dot{\lambda}(t) = h(\lambda(t)) \]
  and thus to a supergradient ascent scheme
  \[ \dot{\lambda}(t) \in \partial F(\lambda(t)) \]
  \( \partial F \) supergradient of \( F \)

- Stability
  - Boundedness of \( \lambda_i \) using projection method or linear stochastic approximation method
Simulations

**Figure:** Convergence for Markovian channel $\alpha = 0.3$, $C = (0.6, 0.8, 0.7, 0.2)$, $\lambda(0) = (1, 1, 1, 1)$ and $\sigma = (1, 1, 0.9, 0.3)$

**Figure:** Gain of the optimal policy over variable power round robin policy, $C=0.6$, $\gamma = 0.7$
Temporal Fairness

- Time as resource
- Different users receive different time resource
  - Long term fair: Proportional share on long run
    - Starvation
    - HOL blocking
  - Short term fair: Proportional share on finite window size $M$
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Temporal Fair scheduler

- **Long term temporal fair**
  - $\phi_i$ temporal share to user $i$ on infinite run
  
  \[
  \lim \inf_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} \mathbb{E} y_i(n) \geq \phi_i \quad \forall i.
  \]

- **Modified update equation**
  
  \[
  \lambda_i(n+1) = \left[ \lambda_i(n) - a(n) \left[ y_i(n) \log \left( 1 + \left( \frac{\lambda_i}{x_i(n)} \right)^+ x_i(n) \right) \right] - C_i \right]^+
  
  \lambda'_i(n+1) = \left[ \lambda'_i(n) - a(n)(y_i(n) - \phi_i) \right]^+ \quad \forall i
  \]

- **Short term temporal fair**
  - $\phi_i$ temporal share on finite window of $M$
  - **Heuristic**: Elimination policy
  Remove the user if its share is exhausted
Simulations

Figure: Trajectory of $\lambda_i(n)$

$\phi = (0.3, 0.4, 0.2, 0.1)$

Figure: Power required for the short term and long term fairness
Preliminaries (2)

Average cost Markov Decision Process:

Path wise Average Cost = \( \limsup_{N \to \infty} E \left[ \frac{1}{N} \sum_{n=0}^{N-1} c(s_n, u_n) \right] \)

\[ \phi^* + h(s) = \min_{u \in U} \left\{ c(s, u) + P(\bar{s}|s, u)h(\bar{s}) \right\} \]

Bellman equation with unique \( h(s) \) exists iff,

\[ h(s) = \min_{u \in U} \left\{ c(s, u) + Eh(\bar{s}) - h(s^0) \right\} \] (5)

where,
- \( s \in S \): State of the process
- \( u \in U \): Action taken
- \( c \): Immediate cost
- \( \bar{s} \): Next state
- \( h \): Difference value function
- \( \phi \): Average cost
- \( s^0 \in S \): Some arbitrary state
- \( P(\bar{s}|s, u) \): Transition matrix (kernel)
Learn how to take action in each state to minimize the cost
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System Model and Problem Formulation

Figure: Transmitter of a point to point link

Optimization problem

Minimize $P = \limsup_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} P_n,$

Subject to

$D - \bar{D} \leq 0$  Delay constraint

$\varepsilon - \bar{\varepsilon} \leq 0^a$  Drop probability constraint

$^a$We denote the constraint by ‘−’.
Figure: System Model

By Little’s law (7) is,

$$Q_{avg} - \bar{a}\bar{D} \leq 0$$

where,

$$\varepsilon = \limsup_{N \to \infty} \frac{\sum_{n=0}^{N} \max(0, Q_n - u_n + a_{n+1} - B)}{\sum_{n=0}^{N} a_n}$$

$$\bar{a} = a_{avg} (1 - \varepsilon) \quad \text{(Effective arrival rate)}$$

$$Q_{avg} = \limsup_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N} Q_n,$$

$$a_{avg} = \limsup_{N \to \infty} \frac{1}{N} \sum_{n=0}^{N} a_n$$
Discrete State Space Markov Decision Formulation: Post Decision

The immediate cost of the constrained MDP is given by,

\[ c_n = P_n + \lambda_1 (\text{Constraint}_1) + \lambda_2 (\text{Constraint}_2) \]

\[ \text{Constraint}_1 : \quad (Q_n - \bar{D}(a_{n+1} - d_n)) \]

\[ \text{Constraint}_2 : \quad (d_n - \bar{\epsilon}a_{n+1}) \]

\(c_n\) is convex function of \(u_n\) and \(s_n\). Hence no duality gap.

- **Post decision state \(\tilde{s}\)**: State after decision is taken

Bellman Equation (5) using the post-decision state is given as,

\[ \tilde{h}_{n+1}(\tilde{s}) = \mathbb{E} \left[ \min_{u \in U} \left\{ c(s, \lambda, u) + \tilde{h}_{n+1}(\tilde{s}|\tilde{s}_n) - \tilde{h}(\tilde{s}^0) \right\} \right] \]

Interchange of \(\min\) and \(\mathbb{E}\) operator
Two time scale update

Faster Time Scale

Value function evaluation

See $\lambda$’s constant

e.g. $\alpha = \frac{1}{n}$

Slower Time Scale

Lagrange Update

e.g. $\beta = \frac{1}{n\log(n)}$

Post learning: Faster time scale

$$\tilde{h}_{n+1}(\tilde{s}) = \tilde{h}_n(\tilde{s}) + \alpha(\nu(\tilde{s}, n))I\{\tilde{s}_n = \tilde{s}\}$$

$$\left[ \min_{u \in U} \left\{ c(s, \lambda_n, u) + \tilde{h}_n(\tilde{s}) \right\} - \tilde{h}_n(\tilde{s}_0) - \tilde{h}_n((\tilde{s})) \right] \forall \tilde{s}$$

Lagrangian update: Slower time scale

$$\lambda_{1n+1} = \Gamma_1[\lambda_{1n} + \beta(n)(Q_n - \bar{D}(a_n - d_n))]$$

$$\lambda_{2n+1} = \Gamma_2[\lambda_{2n} + \beta(n)(d_n - \bar{e}a_n)]$$

$\Gamma_1, \Gamma_2$ are projection operators.
Simulations

Figure: Power delay curve with finite state space
Continuous State Space Formulation

How to deal with large buffer and continuous channel?

Approximate the value function:

\[ h(x) = \sum_{i=1}^{K} f_i(x)r_i, \]

where,
\[ f = [f_1, f_2, \cdots, f_K] : \text{Feature vectors e.g. } [1, Q, x, Qx] \]
\[ r = [r_1, r_2, \cdots, r_K] : \text{Weights} \]
Device an online algorithm to find:
- Quantization $q_k$ per macro block (MB)
- Transmission rate $u_k$ for each MB

and minimize transmission energy subject to distortion and absolute delay constraint per MB
Finite Horizon MDP Optimization Model

Figure: $k^{th}$ MB transmission at $n$th slot

Formulate as finite Horizon MDP with immediate cost:

$$c_n = E_n + \lambda D_n$$
Finite horizon to Infinite Horizon

- Learning algorithms specifically for infinite horizon
- Join horizon $M$ to horizon 0 to get infinite cycle
- Modified learning for finite horizon

Figure: Finite horizon to infinite horizon
Simulation

Figure: Power-Distortion curve
Summary

- Considered joint opportunistic and power optimal solution for minimum rate guarantee
- Proposed power optimal temporal fair scheduling
- Proposed average delay constrained power optimal scheme using MDP formulation.
- Proved convergence of the online policies
- Investigated the issue of power optimal video transmission over wireless
More practical, discrete rate scheduler needs to be designed for minimum rate guarantee
Definition of fairness in fading is an open issue
Convergence of function approximation algorithms using multiple policies is unresolved
Power optimal variable packet length online scheduling algorithm for video transmission can be considered
Thank you
Using ergodicity from (3),

\[
\min \int \nu(dx_1, \cdots dx_N) \sum_{y \in A} \int_{[0,\infty)} p_1(dq|y,x)p_2(y|x)q,
\]

subject to \[
\int \nu(dx_1 \cdots dx_N) \sum_{y \in A} \int_{[0,\infty)} p_1(dq|y,x)p_2(y|x) \log(1 + qy_ix_i) \geq C_i \quad \forall i,
\]

\[q \geq 0. \quad (10)\]

The Lagrange function associated with (10) is,

\[
f(p_1, p_2, \lambda) \triangleq \int \nu(dx_1 \cdots dx_N) \sum_{y \in A} \int_{[0,\infty)} p_1(dq|y,x)p_2(y|x)
\]

\[
\left(q - \sum_i \lambda_i \left[\log(1 + qy_ix_i) - C_i\right]\right). \quad (11)
\]

- \(\nu\) Joint distribution of channel
- \(p_1, p_2\) conditional distributions

Minimizing w.r.t. \(p_1, p_2\),

\[F(\lambda) = \min_{p_1,p_2} f(p_1, p_2, \lambda)\]
Short Term Thoughput Fair

- **Throughput:** $MTC_i$ for window of slot $M$, each slot duration $T$
- **State:** $\{r(n), x(n)\}$
  - $r(n)$ Residual thoughput at slot $n$
  - $x(n)$ Channel state vector
  - $u(n)$ Transmission vector at slot $n$
- **DP formulation**

$$V(n, r(n), x(n)) = \min \left( q(n) + \bar{V}(n + 1, r(n + 1)) \right),$$

$$= \min_{y_i(n), q(n)} \sum_{i=1}^{N} \left( \frac{1}{x_i} \left( e^{u_i(n)y_i(n)} - 1 \right) + \bar{V}(n + 1, r(n + 1)) \right),$$

$$\bar{V}(M + 1, r(M + 1)) = \infty. \quad (12)$$

- **Heuristic:** Elimination policy
Function Approximation

Least Square Policy Iteration:
Minimize the difference between actual and estimated approximated value function by,

\[
\tilde{r}_n = \arg\min_{r} \sum_{m=0}^{n} \left\{ (f(s_m)'r - f(s_m)'r_n - \sum_{k=m}^{n} (\alpha\Lambda)^{k-m}d_n(s_m, s_{m+1}))^2 \right\}
\]

\[
r_{n+1} = r_n + \beta_n (\tilde{r}_n - r_n)
\]

\[
d_n(s_m, s_{m+1}) = c(s_m, s_{m+1}) - \phi_n + (f(s_{m+1}) - f(s_m)r_n, \forall k, n)
\]

\[
\phi_{n+1} = \phi_n + \gamma_n (c(s_n, s_{n+1}) - \phi_n)
\]

- Existence of fixed point proved
- Convergence not proved
Solution using Q Learning

\[ Q_{n+1}(s,u) = Q_n(s,u) + \alpha_n(s,u)e_n \]
\[ \phi_{n+1} = \phi_n + \beta_n e'_n, \]
\[ e_n = \begin{cases} 
  c_n - \phi_n + \max_b Q_n(y_n, b) - Q_n(s,u) & \text{if } (s,u) = (s_n, u_n), x_n \in S_i, i < N \\
  c_n - \phi_n & \text{if } (s,u) = (s_n, u_n), x_n \in S_N \\
  0 & \text{otherwise}
\end{cases}, \]
\[ e'_n = \begin{cases} 
  c_n - \phi_n + \max_b Q_n(y_n, b) - Q_n(s,u) & \text{if } s = s_n, s_n \in S_i, i < N \\
  c_n - \phi_n & \text{if } s = s_n, s_n \in S_N \\
  0 & \text{otherwise}
\end{cases}, \]

(16)