Power Optimal Opportunistic Scheduling in Fading Wireless Channel

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Abstract

In this paper, we propose a power optimal opportunistic scheduling scheme for a multiuser single hop Time Division Multiple Access (TDMA) system. We formulate the problem of minimizing average transmission power subject to minimum rate constraints for individual users. We suggest a stochastic approximation based scheme to implement the policy and prove the convergence and stability of this algorithm. We demonstrate through simulations that the algorithm converges to optimal policy in reasonable iterations for it to be practically useful. Our algorithm is applicable for independent and identically distributed (i.i.d.) as well as Markovian channel fading. Finally, we extend the power optimal scheduling algorithm for providing temporal fairness among users.

Index Terms

Cross Layer Scheduling, Power Efficient, Quality of Service, Stochastic Approximation.

I. Introduction

Next generation wireless networks are likely to provide a unified architecture for the transport of voice, video and data. These applications will be required to be serviced with diverse Quality of Service (QoS) guarantees in terms of performance metrics like throughput, delay, delay jitter, loss and fairness. Bandwidth and power are the two primary resources available to any communication system and hence they must be allocated through appropriate scheduling mechanism to satisfy QoS constraints. Further, in case of wireless networks, where battery and transmission power constraints mandate conservative energy expenditure, the task of energy efficient scheduling becomes very important.

Wireless channel varies with time randomly and asynchronously for different users. This is due to fast multipath fading effects as well as due to different interference levels experienced by users. As a result, *cross layer* resource allocation techniques, i.e. techniques that take into account physical channel characteristics, can achieve significant performance gains. In this paper, we consider such cross layer scheduling algorithms. Specifically, we focus on scheduling algorithms that optimize energy resources subject to QoS contraints like minimum guaranteed rate.

Since wireless channels exhibit time varying fading characteristics, which also vary from user to user, this multiuser diversity can be exploited by *opportunistically* scheduling the user with the best channel condition. Multiuser diversity has been explored in the pioneering work of Knopp and Humblet [1], where the problem of maximizing the information capacity of the uplink in a single cell environment under an average power constraint has been addressed. In this paper also, we consider a single cell multiuser system with Time Divison Multiple Access (TDMA). We assume a block fading channel model where the channel remains constant over each slot duration, however, different users experience different channel conditions. While, *pure* opportunistic scheduling would schedule the user with the best channel condition, we also consider energy efficiency. Specifically, the scheduling algorithm proposed in this paper determines the user to be scheduled in each time slot so that the overall average transmission power is minimized to meet minimum rate constraint of each individual user. In that sense, the problem considered in this paper is a dual of [1]. Moreover, we have also given an on-line algorithm based on stochastic approximation and prove the convergence of this algorithm for both independent and identically distributed (i.i.d) and Markov channel fading. Finally, we have extended our framework to incorporate fairness constraints also.

A. Related Work

There is a considerable literature on energy efficient scheduling [2], [3], [4]. These papers have considered the problem of minimizing average power subject to a constraint on the average delay. They have also investigated various structural properties of optimal policy and quantified the tradeoff between delay and power. A comprehensive review of energy efficient scheduling has been given in [5]. However, all these works have considered the problem of scheduling the packets over a point to point wireless channel. In this paper, we consider a multiuser scenario where the users can be scheduled opportunistically. Opportunistic scheduling, however, introduces the issue of fairness among users. Since the user with the best condition always transmits, some users who continue to experience bad channel may starve for a long time. Thus, there is a tradeoff between total system throughput and fairness among users. The concept of proportional fairness in multiuser diversity has been investigated in [6]. The proportional fair scheduler can be shown to be maximizing the logarithmic utility function for the users in asymptotic sense. Variants of proportional fair schedulers like Modified Weighted Delay First (M-LWDF) strategy has been introduced in [7]. In [8], the author points out that the proportional fair scheduler is not always stable. A modified fair rule in [9] called exponential rule is able to provide stability, if there exists any feasible policy which can achieve stability.

The problem of fainrness in opportunistic scheduling has also been investigated in [10], [11], [12] under various types of fairness measures. In [10], an optimal index policy has been proposed for long term fairness in terms of bandwidth allocation. In [11], the authors consider the problem of throughput maximization with deterministic and probabilistic long term fairness constraints. As apposed to throughput fairness, an alternate notion of temporal fairness may be more appropriate for wireless opportunistic schedulers. By temporal fairness, we mean that each user has access to certain number of time slots. In [12], the authors study scheduling policies under Short Term Temporal Fairness (STF) constraints. Short term temporal fairness reduces the inter scheduling delays at the cost of throughput. Using special case of window size of $M = N$ and $M = \infty$, where N is the number of users, the STF constrained policy assigns $\phi_i M$ number of time slots to a user i in any scheduling frame of window M and maximizes the system throughput under these constraints where ϕ_i is the weight assigned to the user i such that $\sum \phi_i \leq 1$. A heuristic policy that maximizes the system throughput, while trying to satisfy the required STF constraints has been suggested. It has been proved that such allocation in opportunistic regime gives more throughput than scheduling non-opportunistically.

We observe that while energy efficient scheduling has been studied on a point to point channel, the focus of multiuser scheduling has been largely on opportunistic scheduling under fairness constraints. Our work is different from all these works in the sense that we introduce energy efficient scheduling algorithm for single cell multiuser opportunitsic setting. In [13], the authors have considered an interference-based joint scheduling and power allocation scheme for a multi-cellular environment. Though their problem setting is different, the mathematical formulation is similar to ours. However, issues such as the convergence, optimality and stability of the iterative algorithm have also not been addressed. Moreover, we have validated our algorithm for i.i.d. as well as Markovian channel fading. Further, our formulation also incorporates temporal fairness.

The rest of the paper is organized as follows. In Section II and III, we describe our system model and derive the optimal scheduling policy. In Section IV, we describe the stochastic approximation method used to implement the opportunistic power optimal scheduling policy. We compare the performance of this scheme with power optimal round robin and ‘best channel first’ algorithm. In Section V, we introduce a temporal short term fair power optimal scheme and present the simulation results. Finally, we conclude the paper in Section VI

II. System Model

We consider a multiuser TDMA system with base station as the centralized scheduler. Time is divided into slots of equal duration. The channel is assumed to be time varying with block fading model, i.e. channel is constant over a slot duration and changes only at slot boundaries. In any given time slot, only one user is allowed to transmit. If ith user transmits in slot n, then the received signal is given by

$$r(n) = \sqrt{x_i(n)}s(n) + w(n) \tag{1}$$

where $s(n)$ is the transmitted signal (with power $q_i(n)$), $w(n)$ is zero mean Additive White Guassian noise of power spectral density $N_0/2$ and $\sqrt{x_i(n)}$ is the time varying channel gain of ith user due to fading. Let W denote the received signal bandwidth, then the received signal to noise ratio is $SNR = q_i(n)x_i(n)/N_0W$. In our further discussion, we assume N_0W to be unity, i.e. we assume that the SNR is normalized to the noise power.

Each user experiences a different channel gain. The channel state at the beginning of slot n is denoted by the vector $(x_1(n), x_2(n), \cdots, x_N(n))$, where $x_i(n)$ corresponds to the channel for user i at slot n and N is the number of users. We assume that perfect channel state information (CSI) is available to the scheduler.

The channel state process $(x_1(n), x_2(n), \cdots, x_N(n))$ is assumed to be $\mathbb{R}^d-$valued and ergodic with marginal distribution ν, where $N \geq d \geq 1$. Due to ergodicity, henceforth, we do not explicitly state the dependence of the channel state process on time n. Thus, let $\mathbf{x} = (x_1, \cdots, x_N)$ denote the channel state vector.

The channel gains experienced by different users are assumed to be i.i.d. The channel state evolution with time can be either i.i.d. or Markovian. Under the Rayleigh fading model, each x_i is exponentially distributed, i.e. the probability density function of x_i is given by

$$\mu(x_i) = \frac{1}{2\sigma_i^2} \exp^{\frac{-x}{2\sigma_i^2}}$$

We assume that each user can specify its QoS constraint in terms of certain minimum required utility which may be a function of the transmission power and the channel gain. In a practical system, this utility could be defined in terms of minimum guaranteed data rate or throughput which not only depends on the transmitted signal power and the channel gain but also on the desired bit error rate and the modulation and coding parameters. In this paper, we do not assume any specific modulation or coding scheme. Accordingly, we consider the rate to be the information theoretic rate $U_i(q_i, x_i) = \log(1 + q_i x_i)$ in bits/sec/Hz. Note that our formulation in this paper is applicable to any utility function U_i which is increasing and concave in channel gain x_i and power q_i. We assume that this rate requirement of each user is known a-priori to the scheduler.

III. Optimal Scheduling

In this section, we formulate the problem of power optimal opportunistic scheduling. Since, we have assumed the system to be TDMA, only one user transmits in a slot and the scheduler determines the user who can transmit. The scheduler also determines its transmission power subject to that user's rate constraint. For each user i, we associate an indicator function $y_i(n)$ which is 1 if user i is scheduled at time slot n, otherwise it is 0. Let $q(n)$ be the actual transmission power of the scheduled user at time slot n. (Here, we have dropped the index i from $q(n)$. Thus $q(n)$ corresponds to any user that was scheduled in slot n). Let C_i be the time-average minimum rate requirement for user i. Our objective is to minimize average power subject to the rate constraints, which can be expressed as:

$$\min \limsup_{M \to \infty} \frac{1}{MN} \sum_{n=1}^{M} \sum_{i=1}^{N} q(n) y_i(n),$$

$$\text{s.t.} \liminf_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} U_i(y_i(n) q_i(n), x_i(n)) \geq C_i \; \forall i \tag{2}$$

where $U_i(y_i(n)q(n), x_i(n)) = \log(1 + y_i(n)q(n)x_i(n))$. Let $A = (\mathbf{e_1}, \cdots \mathbf{e_N})$, where $\mathbf{e_i}$ denotes the unit vector in the i^{th} coordinate direction. Let $\mathbf{y} = (y_1, \cdots, y_N)$ be the vector of indicator random variables. Note that only one of the random variables y_i will be 1 in a given time slot. Let p be the conditional law of $(q, \mathbf{y})$ given $\mathbf{x}$, which can be decomposed as $p_1(dq|\mathbf{y}, \mathbf{x}) p_2(\mathbf{y}|\mathbf{x})$. Thus, we can write the optimization problem (2) as:

$$\min \int \nu(dx_1, \cdots, dx_N) \sum_{\mathbf{y} \in A} \int_{[0,\infty)} p_1(dq|\mathbf{y}, \mathbf{x}) p_2(\mathbf{y}|\mathbf{x}) q,$$

$$\text{s.t.} \int \nu(dx_1, \cdots, dx_N) \sum_{\mathbf{y} \in \mathbf{A}} \int_{[0,\infty)} p_1(dq|\mathbf{y}, \mathbf{x}) p_2(\mathbf{y}|\mathbf{x})$$

$$\log(1+qy_ix_i) \geq C_i \;\; \forall i, \;\; q \geq 0. \tag{3}$$

Proposition 1: The optimal policy is to select user k and transmission power q^*, where

$$\begin{aligned} k &= \arg\min_i \left\{ \left(\lambda_i - \frac{1}{x_i}\right)^+ - \lambda_i \Bigg[\right. \\ &\quad \left. \log\left(1 + \left(\lambda_i - \frac{1}{x_i}\right)^+ x_i\right) - C_i \Bigg] \right\}, \end{aligned} \tag{4}$$

$$q^* = \left(\lambda_k - \frac{1}{x_k}\right)^+, \tag{5}$$

and λ_i is the Lagrange multiplier associated with the rate constraint for user i.

Proof: The Lagrangian associated with (3) is

$$\begin{aligned} f(p_1, p_2, \boldsymbol{\lambda}) \triangleq \int \nu(dx_1, \cdots, dx_N) \sum_{\mathbf{y}\in A} \int_{[0,\infty)} p_1(dq|\mathbf{y}, \mathbf{x}) \\ p_2(\mathbf{y}|\mathbf{x}) \left(q - \sum_i \lambda_i \left[\log\left(1 + qy_ix_i\right) - C_i\right] \right) \end{aligned} \tag{6}$$

where $\boldsymbol{\lambda} = (\lambda_1, \cdots, \lambda_N)$. Therefore, the optimization problem decomposes into: minimize with respect to (w.r.t.) $p_1(q|\mathbf{x}, \mathbf{y})$ and then minimize w.r.t. $p_2(\mathbf{y}|\mathbf{x})$. Note that the cost function $f(p_1, p_2, \boldsymbol{\lambda})$ is linear in the joint probability distribution when the marginal distribution of $\mathbf{x}$ is fixed and the minimization is over the conditional distributions. The set of probability distributions with a fixed marginal is a closed convex set with extreme points corresponding to those distributions for which the conditional distributions are point masses [14]. Thus for each $\mathbf{x}$, we minimize over q and $\mathbf{y}$. The Lagrangian (6) is strictly convex w.r.t. q and $\mathbf{y}$ and hence the minimizer is unique. Since joint minimization over q and $\mathbf{y}$ can be done in any order, we minimize first with respect to q and then w.r.t. $\mathbf{y}$. Thus we first minimize (6) w.r.t. q for a fixed i which corresponds to $\mathbf{y} = \mathbf{e}_i$. The reduced single user min-max problem is:

$$\max_{\lambda_i} \min_q \mathcal{L}(\lambda_i, q) \tag{7}$$

where $\mathcal{L}(\lambda_i, q) = q - \lambda_i(\log(1+qx_i) - C_i)$. Denote the optimal q for $\mathbf{y} = \mathbf{e}_i$ by q_i^*. To minimize (7) w.r.t. q, we differentiate $\mathcal{L}(\lambda_i, q)$ w.r.t. q,

$$\frac{\partial \mathcal{L}}{\partial q} = 1 - \lambda_i \left(\frac{x_i}{1+qx_i}\right), \tag{8}$$

leading, by the Kuhn-Tucker theorem [15], to

$$q_i^* = \left(\lambda_i - \frac{1}{x_i}\right)^+ . \tag{9}$$

Minimizing (6) w.r.t. $\mathbf{y}$ yields the optimal policy,

$$\begin{aligned} k &= \arg\min_i \left\{ q_i^* - \lambda_i \left[\log(1 + q_i^* x_i) - C_i\right] \right\} \\ &= \arg\min_i \left\{ \left(\lambda_i - \frac{1}{x_i}\right)^+ - \lambda_i \left[\right.\right. \\ &\qquad \left.\left. \log\left(1 + \left(\lambda_i - \frac{1}{x_i}\right)^+ x_i\right) - C_i \right] \right\}. \end{aligned} \tag{10}$$

The optimal policy is to schedule user k which satisfies (10). The scheduled user will transmit with power q^* as given in (9) with λ_i and x_i replaced by λ_k and x_k. Thus $q^* = \left(\lambda_k - \frac{1}{x_k}\right)^+$ ■

To implement the above policy, the Lagrange multipliers $\boldsymbol{\lambda}$ need to be computed. These can be determined such that the constraints in (3) are saisfied. The computation of optimal policy would also require the knowledge of the distribution of the channel state vector $\mathbf{x} = (x_1, \cdots, x_N)$. Though under usual assumption of Rayleigh fading model, x_i is exponentially distributed, the knowledge of this distribution may not be known in practice. It is thus useful to determine an on-line algorithm to implement the optimal policy.

IV. On-line Algorithm based on Stochastic Approximation

In this section, we propose an on-line algorithm to estimate parameters $\boldsymbol{\lambda}$ of the policy based on stochastic approximation. The policy and the update equation involved in the algorithm are low in complexity. The stochastic approximation algorithm guarantees almost sure (a.s.) convergence to the optimal solution, if certain properties of the update equation and the objective functions are satisfied. We prove that these properties are indeed satisfied in our case and thus the algorithm converges to optimal $\boldsymbol{\lambda}$ with probability (w.p.) 1. Stochastic approximation can be used to determine an optimum solution for a perturbed function (in our case perturbation is channel fading). After minimizing (6) over $(q, \mathbf{y})$ in Section III, we maximize over $\boldsymbol{\lambda}$ to obtain the optimal solution.

We propose the following on-line update equation for computing $\boldsymbol{\lambda}$. Note that this is a stochastic gradient ascent scheme.

$$\lambda_i(n+1) = \Gamma\Bigg(\lambda_i(n) - \alpha(n)\Bigg[y_i(n)\log\Bigg(1 + \left(\lambda_i - \frac{1}{x_i(n)}\right)^+ x_i(n)\Bigg)\Bigg] - C_i\Bigg)\forall i \quad (11)$$

where[1]:

1) $y_i(n) = I(q_i^* - \lambda_i\left[\log(1+q_i^* x_i) - C_i\right] \leq (q_j^* - \lambda_j[\log(1+q_j^* x_j) - C_j]), j \neq i$.
2) $\alpha(n)$ is a positive scalar sequence satisfying [16],

$$\sum_n \alpha(n) = \infty, \quad \sum_n \alpha(n)^2 < \infty,$$

3) $\Gamma(\cdot)$ is the projection to the set $[0, L]$ where $L \geq 0$ is a very large but finite number, i.e., $\Gamma(x) = \max(0, \min(x, L))$.
4) We take $\alpha(n) = \frac{l}{n}$, where l, the initial learning rate, is a small constant.

Note that we have assumed the transmission power q to be unconstrained. However, if we impose a constraint $q \leq q_{max}$ for a prescribed $q_{max} < \infty$, then we can replace q^* by $\hat{q}^* = q^* \wedge q_{max}$[2]. In [10], [11], [12] also, the authors have used stochastic approximation algorithm, but the convergence proof is not discussed. Moreover the technical proof in these algorithms is simple because of the differentiable functions involved. These assumptions are not applicable here.

We now sketch the proof of convergence for the stochastic approximation scheme as outlined in (11). The details are discussed in Appendices I and II. We consider the channel state process to be i.i.d. across slots. The proof of convergence for the Markovian model is along similar lines.

Let $\tilde{y}_i(n) = y_i(n)$ with $\lambda_i(n)$ replaced by λ_i and $E_s[\,\cdot\,]$ denote the stationary expectation. Rewrite iteration (11) as,

$$\lambda_i(n+1) = \Gamma\left(\lambda_i(n) - \alpha(n)\left[h_i\left(\boldsymbol{\lambda}(n)\right) + M_i(n+1)\right]\right), \quad (12)$$

[1] $I(a \leq b) = 1$ if $a \leq b$, $= 0$ otherwise.
[2] $a \wedge b = \min(a, b)$

where,

$$\begin{aligned} h_i(\boldsymbol{\lambda}(n)) &= E_s\Bigg[\tilde{y}_i(n)\Bigg(\log\left(1+\left(\lambda_i - \frac{1}{x_i(n)}\right)^+ x_i(n)\right) \\ &\quad -C_i\Bigg)\Bigg] \;|_{\lambda_i=\lambda_i(n)} \\ M_i(n+1) &= y_i(n)\log\left(1+\left(\lambda_i(n) - \frac{1}{x_i(n)}\right)^+ x_i(n)\right) \\ &\quad -C_i - h_i\left(\boldsymbol{\lambda}(n)\right). \end{aligned}$$

This iteration will converge w.p. 1 to an invariant set of the differential equation,

$$\dot{\boldsymbol{\lambda}}(t) = \mathbf{h}(\boldsymbol{\lambda}(t)) + \mathbf{z}(t), \tag{13}$$

where $\mathbf{h}(\cdot) = [\mathbf{h_1}(\cdot), \cdots, \mathbf{h_N}(\cdots)]$ and $\mathbf{z}(t) = [z_1(t), \cdots, z_N(t)]$ is the boundary correction term due to the projection operator Γ [16]. Note that $h_i(\boldsymbol{\lambda}) \in \partial F(\boldsymbol{\lambda})$, where,

$$\begin{aligned} F(\boldsymbol{\lambda}) &= E_s\Bigg[\min_i \Bigg\{ \left(\lambda_i - \frac{1}{x_i(n)}\right)^+ - \lambda_i\Bigg(\log\Bigg(\\ &\quad 1+\left(\lambda_i - \frac{1}{x_i(n)}\right)^+ x_i(n)\Bigg) - C_i\Bigg)\Bigg\}\Bigg] \end{aligned} \tag{14}$$

is the point-wise minimum of a family of affine functions of $\boldsymbol{\lambda}$ and is a strictly concave function of $\boldsymbol{\lambda}$. ∂F denotes its superdifferential. Thus the ordinary differential equation (13) may be viewed as the differential inclusion

$$\dot{\boldsymbol{\lambda}}(t) \in \partial F(\boldsymbol{\lambda}(t)) + \mathbf{z}(t). \tag{15}$$

The proof details for this are given in Lemma 3 of Appendix I. Note that (15) is a supergradient ascent scheme for a strictly concave function and thus will converge to its unique maximum on the constraint set. If L is sufficiently large, this will be the desired vector of Lagrange multipliers by the saddle point theorem [15]. Thus, the iterates (11) converge almost surely to the Lagrange multipliers.

A. Simulation Results

We first perform the simulations to demonstrate the convergence of $\lambda_i(n)$. Consider a single cell network of 4 wireless users with 1 base station. We assume a Rayleigh fading channel, where the probability density function of x is given by $\mu(x) = \frac{1}{2\sigma^2}e^{\frac{-x}{2\sigma^2}}$, where $\sigma > 0$. Each user experiences a different channel gain x according to this density function with different values of σ. For 4 users, we choose the values of σ to be $0.6, 0.8, 1, 1.2$. We first consider the case of i.i.d. fading. Thus, each user observes the channel in a slot independent of the other slots. We also assume the rate constraints C_i for 4 users to be $1.0, 1.0, 1.5, 1.5$ bits/sec/Hz respectively. We consider a slotted TDMA system. In each slot, every user updates its λ using stochastic approximation algorithm (11). In this algorithm, we assume $\alpha(n) = \frac{l}{n}$. We choose $l = 10$. We begin our simulations using the initial values of λ for 4 users to be $\boldsymbol{\lambda}(0) = (1, 1, 1, 1)$. The user and its tarnsmission power is selected using optimal policy (10). Figure 3 shows the convergence for i.i.d. channel. The average powers (normalized to thermal (additive white guassian) noise power) required by the four users to achieve the desired rates over 10 independent runs are $4.97, 3.74, 4.91, 4.20$ dB respectively.

We next model the more general case of Markovian channel fading to demonstrate the correctness of our algorithm. In Markovian channel fading model, we assume that the channel gain for user i obeys the auto-regressive equation,

$$x_i(n+1) = \beta x_i(n) + (1-\beta)g_i(n), \tag{16}$$

where noise $g_i(n)$ is zero mean Gaussian with variance σ (we assume same σ for all users) and α is correlation coefficient. Note that $x_i(0)$ is exponentially distributed with $\mu(x) = \frac{1}{2\sigma^2}e^{\frac{-x}{2\sigma^2}}$. We take $\beta = 0.3$. The remaining paramters for the simulation are the same as above. Figure 4 shows a particular trajectory for $\lambda_i(n)$. Our results demonstrate that the Lagrange multipliers converge for all users within 3000 iterations. The normalized average powers required for 4 users are given by $7.67, 6.42, 7.61, 6.88$ dB respectively. From these values, it can be inferred that the average power required for the Markovian channel is greater than that of i.i.d. channel case, as expected.

Remark 1: In wireless data transfer applications, the duration of transfer is of the order of seconds, while the slot duration is of the order of microseconds. Hence, even if there is non-optimality for the initial slots, our results demonstrate that convergence will occur much before the actual completion of data transfer.

Remark 2: In practical scenarios, we may not want actual convergence to take place, or we may only like to be within the neighborhood of the optimal solution. In [17], lock-in phenomenon for stochastic approximation algorithm has been considered. If the iterate $\boldsymbol{\lambda}(n)$ is within the domain of attraction (the iterate has begun to converge), then there exists a finite number of iterations for the iterate to be within a finite distance from the convergence point $\boldsymbol{\lambda}^*$. A probabilistic lower bound is given for the occurrence of this “nearness” within finite number of iterations.

1) Comparison with Round Robin Scheme: We now compare our scheduling policy with the round robin scheme with optimal power transmission. For these simulations, we consider a symmetric system, i.e., all users have the same channel statistics, i e. the density function of x is exponential with same σ for all users. All users also have the same minimum rate constraints. For simulation purposes, we assume $C = 1$ bits/sec/Hz and $\sigma = 1$. We simulate the system with our policy in a manner similar to as described above. We perform the simulations for varying number of users but each user with the same minumum rate constraint. For each case, we compute the average transmission power requirement.

In the round robin scheme, each user is selected to transmit in a slot in a round robin fashion. The user transmits with an optimal power. To determine the optimal power, we consider an equivalent single user point to point system with minimum rate guarantees equal to NC. Thus we can determine the power p such that

$$\int \log(1 + p(x)x)\mu(x)\, dx = NC \tag{17}$$

is satisfied.

We compare the average power required in the proposed optimal policy with round robin scheme with optimal power. The results, shown in Figure 5, demonstrate that as the number of users increases, the ratio of average transmission power of the optimal policy to that of the round robin policy increases, but the marginal increase per user decreases. The gain obtained from power optimal opportunistic policy with number of users is due to multiuser diversity.

We have also simulated the system with different values of σ and it has been observed that this ratio remain nearly the same.

2) Comparison with Best Channel First scheme: We now compare our scheme with opportunistic scheme where the user with the best channel transmits. We simulate the scheme for 8 users and assume the values of σ to be equal to $1.6, 1.6, 1.2, 1.2, 1, 1, 0.8, 0.8$ respectively. We assume C to be 1 bits/sec/Hz for all users. For the proposed optimal power scheme, the normalized average powers required for 8 users are $9.91, 9.91, 10.2, 10.2, 10.5, 10.5, 10.8, 10.8$ dB and the total average power is dB. For the opportunistic scheme, we schedule the user with the best channel condition with a power so that the average rate constraint is satisfied. The average powers required are $2.3, 2.3, 9.8, 9.8, 20.9, 20.9, 38.8, 38.7$ dB and the total average power is 41.7 dB. Thus we note that not only the total system power requirement for best channel first scheme is more than that of the power optimal policy, even the individual user's power requirement is also more. We observe that the proposed policy achieves some kind of power fairness.

V. Temporal Fairness

In this section, we introduce fairness while considering the power optimization. The power optimal scheduling scheme considered in Section III may result in starvation of strong users in order to satisfy the rate guarantees of weak users. Hence, in this section, we develop a scheduling algorithm which minimizes power while providing short term fairness and minimum rate guarantees. We first propose a long term fair scheduler and then propose a heuristic short term fair scheduler.

A. Long Term Fairness

Our objective is to opportunistically schedule the user with the best channel condition such that rate guarantees and temporal fairness are achieved and the average transmission power is minimized.

Let ϕ_i be the proportion of *temporal bandwidth* allocated to user i and $\boldsymbol{\phi} = [\phi_1 \phi_2 \cdots \phi_N]$. Thus, ϕ_i represents the fraction of the time slots allocated to user i. Our objective is to minimize average power subject to rate and fairness constraints. The optimization problem is the same as that of (2) with the following additional constraint

$$\liminf_{M \to \infty} \frac{1}{M} \sum_{n=1}^{M} \mathbf{E}\, y_i(n) \geq \phi_i \quad \forall i. \tag{18}$$

Using the ergodicity assumption from Section III, the Lagrangian with the fairness constraint is:

$$f(p_1, p_2,\ \boldsymbol{\lambda}) \triangleq \int \nu(dx_1, \cdots, dx_N) \sum_{y \in A} \int_{[0,\infty)} p_1(dq|\mathbf{y}, \mathbf{x})$$
$$p_2(\mathbf{y}|\mathbf{x}) \left(q - \sum_i \lambda_i \left[\log(1 + q y_i x_i) \ \ - C_i\right] + \sum_i \lambda_i'(y_i - \phi_i) \right),$$

where λ_i' is the Lagrange multiplier associated with the constraint (18), $\boldsymbol{\lambda}$ is the vector $(\lambda_1, \cdots, \lambda_N, \lambda_1', \cdots, \lambda_N')$. Following the approach adopted in Section III, we obtain the optimal policy as: Select user k and transmission power q^* where

$$\begin{aligned} k &= \arg\min_i \Big\{ \Big(\lambda_i - \frac{1}{x_i}\Big)^+ - \lambda_i \Big[\log\Big(\\ &\qquad 1 + \Big(\lambda_i - \frac{1}{x_i}\Big)^+ x_i\Big) - C_i\Big] + \lambda_i'(1-\phi_i)\Big\} \qquad (19) \\ q^* &= \Big(\lambda_k - \frac{1}{x_k}\Big)^+. \qquad (20) \end{aligned}$$

Using the stochastic approximation algorithm from Section III, the Lagrange multiplier update equations can be written as

$$\begin{aligned} \lambda_i(n+1) &= \Big[\lambda_i(n) - a(n)\Big[y_i(n)\log\Big(\\ &\qquad 1 + \Big(\lambda_i - \frac{1}{x_i(n)}\Big)^+ x_i(n)\Big)\Big] - C_i\Big]^+ \\ \lambda_i'(n+1) &= [\lambda_i'(n) - a(n)(y_i(n) - \phi_i)]^+ \quad \forall i \qquad (21) \end{aligned}$$

The optimality of the above scheme can be proved in a manner similar to that in Section III.

B. Short Term Fairness

In Section V-A, we have considered long term fairness. Long term fairness guarantees average proportional time share. However, one of the problems associated with long term fairness is starvation or Head of Line (HOL) blocking. There exist conditions when a user may not get a chance to transmit for some period of time even after being assured a minimum rate guarantee. Thus, it is important to consider a short term fair scheduler.

We consider a window of size $M \geq N$ slots. In a short term fair scheduler, we allocate time share equal to $\phi_i M$[3] to user i over this window and say that the scheduler is short term fair over the window M. The case $M \to \infty$ is same as the long term fairness. We first discuss the case when $M = N$. Let $\mathcal{A}$ be the set of users, i.e., user $k \in \mathcal{A}$. For $M = N$, we can allocate a maximum of one slot per user. We first select the user from the set $\mathcal{A}$ which is optimal for that time slot from (19)[4]. Let k be the optimal user. We remove user k from the list: $\mathcal{A} = \mathcal{A} \setminus \{k\}$. We repeat the above process on modified $\mathcal{A}$. We call this policy as *elimination policy*. The algorithm for general M is explained below.

[3] We assume that $\phi_i M$ is an integer $\forall\, i$.

[4] In the modified algorithm, the set of users is $\mathcal{A}$.

Algorithm 1 Temporal Short Term Fair Scheduling

Slot vector **v**= $M(\phi_1, \phi_2, \cdots, \phi_N)$
$\mathcal{A} = \{1, 2, \cdots, N\}$
$i = 1$
for $i \leq M$ **do**
    for each $j \in \mathcal{A}$ **do**
        Choose optimal k using (19).
        Transmit with power q^*
        $\{\mathbf{v}\}_k = \{\mathbf{v}\}_k - 1$
        if $\{\mathbf{v}\}_k \leq 0$ **then**
            $\mathcal{A} = \mathcal{A} \setminus \{k\}$
        end if
    end for
end for

C. Simulation Results

We perform the simulations in a manner similar to Section IV-A except that we consider the fairness constraints as well. We assume the rate constraints for 4 users to be $1, 1, 1.5, 1.5$ bits/sec/Hz and the fairness constraints ϕ to be 0.25 for all users. In our simulations, we assume that the channel gains are Markovian across slots, as in (16). In (16), we consider the values of σ for 4 users to be $0.6, 0.8, 1.0, 1.2$ respectively. We perform the simulation over 10000 time slots. In Figure 6 we have shown a snapshot of a particular trajectory for $\lambda_i(n)$. The results demonstrate that the λs converge for all users. The average power to achieve the desired rates over 50 independent runs for 4 users are $13.09, 11.71, 15.32, 14.64$ dB.

We also plot the average power required with increasing window size in Figure 7. The power required is a decreasing function of the window size. The actual fairness achieved by the long term and short term temporal fair algorithm are plotted in Figure 8. It may be noted that short term fair scheduler is not optimal. In this scheme, more emphasis is given to providing temporal fairness, but in the process, the actual rate obtained may deviate from the desired rates. Thus there is tradeoff between window size and the achieved rates.

VI. Conclusions

In this paper, we have obtained a power optimal opportunistic scheme for multiuser TDMA system with minimum rate constraints for individual users. We have proposed an online optimal scheduling algorithm based on stochastic approximation and proved the convergence of the algorithm to optimal policy. We have demonstrated the superiority of our algorithm over Round Robin Scheme with optimal power and Opportunsitic Scheme with Beest Channel First through simulation results. We have performed the simulations for both i.i.d. as well as correlated fading model. Finally, we have extended the approach to incorporate temporal fairness constraints as well and proposed a heuristic based short term fair algorithm. We have compared the performance of the heuristic algorithm with long term fair algorithm. This framework can be extended to multihop TDMA network. We can also take into account delay constraint of users. We are currently investigating in these directions.

Appendix I
Existence of optimal stochastic approximation algorithm

In this appendix, we prove the convergence of the stochastic approximation scheme given in (11).

Let $F(\boldsymbol{\lambda}) \stackrel{\Delta}{=} \min_{p_1,p_2} f(p_1, p_2, \boldsymbol{\lambda})$. Let $D_x F$ denote the partial differentiation of F w.r.t. x. The differential inclusion of F at $\boldsymbol{\lambda}$ is denoted by $\partial F(\boldsymbol{\lambda})$. Note that ∂F is upper semicontinuous [18]. For the existence of optimal solution for (3), we must have, stationary point $0 \in \partial F$.

Note that we have used a projection operator in the stochastic approximation algorithm (11), however, we present the proof here without projection. The proof for the convergence with projection operator can be given by extending this proof with the techniques given in [16].

Lemma 1:

$$F(\boldsymbol{\lambda}) \text{ is concave.}$$

Proof: $F : \{0, \mathbb{R}^+\} \rightarrow \mathbb{R}$ is affine in $\boldsymbol{\lambda}$ and F is the point-wise minimum of a family of affine functions of $\boldsymbol{\lambda}$. Hence F is concave. ■

Lemma 2: The stochastic approximation scheme for the maximization of function $F(\boldsymbol{\lambda})$ is given by,

$$\boldsymbol{\lambda}(n+1) = (\boldsymbol{\lambda}(n) - \alpha(n)[\mathbf{h}(\boldsymbol{\lambda}(n)) + \mathbf{M}(n+1)]),$$

provided $\mathbf{h}(\boldsymbol{\lambda}) \in \partial F$. Here, $\mathbf{M}(n+1) = (M_1(n+1), M_2(n+1) \ldots, M_N(n+1))$ where $M_i(n+1)$ has been defined in (12).

Proof: F is a concave function by Lemma 1. A concave function is continuous in the interior of the domain. Hence F is continuous over $(0, \infty)$. By stochastic subgradient descent algorithm [19], Lemma 2 is proved. ■

Lemma 3:

$$\dot{\boldsymbol{\lambda}}(t) \in \partial F(\boldsymbol{\lambda}(t)). \tag{22}$$

Proof: We note that $f(., p_1, p_2)$ is affine, continuous and differentiable in $\boldsymbol{\lambda}$ and continuous in p_1, p_2. Hence the following properties are satisfied-

1) $f(\boldsymbol{\lambda}, p_1, p_2)$ is differentiable at $\boldsymbol{\lambda}$ uniformly in p_1, p_2,
2) $D_{\boldsymbol{\lambda}} f(\boldsymbol{\lambda}, p_1, p_2)$ is continuous in p_1, p_2,
3) $f(\boldsymbol{\lambda}, p_1, p_2)$ is lower semicontinuous in $\boldsymbol{\lambda}$.

If the above conditions are satisfied, then from [20], it can be shown that $\partial F(\boldsymbol{\lambda}) = \bar{co}Y(\boldsymbol{\lambda})$ where $\bar{co}Y$ denotes the compact convex hull of Y and $Y(\boldsymbol{\lambda}) := \left\{D_{\boldsymbol{\lambda}} F(\boldsymbol{\lambda}) \forall p_1, p_2 \text{ that minimizes} f(\boldsymbol{\lambda}, p_1, p_2)\right\}$.

Let $S : \mathbb{R}^N \rightarrow \mathbb{R}^N$ denote a set-valued map satisfying the following conditions:

1) S is upper semicontinuous,
2) For each $g \in \mathbb{R}^N, S(g)$ is convex and and compact set,
3) For some $K > 0$ and for all g in each bounded ball $\mathcal{B} \in \mathbb{R}^{\mathcal{N}}$,

$$\sup_{s \in S(g)} ||s|| < K(1 + ||g||),$$

where $||\ ||$ denote the Euclidean norm on $\mathbb{R}^N$.

With these general conditions, it can be proved [19] that the stochastic approximation of the following form,

$$g(n+1) = g(n) + \alpha(n) \left[s(n) + M(n+1)\right], \quad s(n) \in S(g), \tag{23}$$

will characterize a stochastic inclusion limit $\dot{g}(t) \in S(g(t))$.

The set valued map ∂F satisfies the above properties 1 and 2 by the definition of superdifferential [18]. Let us restrict $s(n)$ to have values $\mathbf{h}(\boldsymbol{\lambda}(n))$. We now proceed to prove $||\mathbf{h}(\boldsymbol{\lambda})|| \leq$

$K(1+||\boldsymbol{\lambda}||)$ and $\mathbf{h}(\boldsymbol{\lambda}) \in \partial F$. To prove this, first consider

$$\begin{aligned}
&\int [\nu(dx) y_i(n) \log(1 + (\lambda_i(n) - \frac{1}{x_i})^+ x_i) - C_i] \\
&= [\int \nu(dx) \left(y_i(n) \log(1 + (\lambda_i(n) - \frac{1}{x_i})^+ x_i) \right)] \\
&\leq [\int \nu(dx) \left(\log(\lambda_i(n) x_i) \right)] \\
&\leq \int \nu(dx) \log(\lambda_i(n)) + \int \nu(dx) \log(x_i) \\
&< \hat{K}(1+\lambda_i) \text{ for some } \hat{K}
\end{aligned} \tag{24}$$

From (24) and definition of $||\mathbf{h}(\boldsymbol{\lambda})||$ we get,

$$||\mathbf{h}(\boldsymbol{\lambda})|| < K(1+||\boldsymbol{\lambda}||). \tag{25}$$

Now we have to show $\mathbf{h}(\boldsymbol{\lambda}) \in Y(\boldsymbol{\lambda})$. Consider $f(\boldsymbol{\lambda}, p_1, p_2)$ where only $y_i(n) = 1$. Differentiating this f w.r.t. λ_i and substituting optimal value of q^* we get,

$$\begin{aligned}
\left.\frac{\partial f}{\partial \lambda_i}\right|_{q^*} &= -[\int \nu(dx) \left(y_i(n) \log(1 + (\lambda_i(n) - \frac{1}{x_i})^+ x_i) \right) - C_i]. \\
&\Rightarrow \mathbf{h}(\boldsymbol{\lambda}) \in \bar{co} Y(\boldsymbol{\lambda}).
\end{aligned}$$

Since $\partial F(\boldsymbol{\lambda}) = \bar{co} Y(\boldsymbol{\lambda})$, then $\mathbf{h}(\boldsymbol{\lambda}) \in \partial F(\boldsymbol{\lambda})$. Thus if we consider the set valued map S to be ∂F, then stochastic approximation (22) satisfies the differential inclusion limit $\dot{\boldsymbol{\lambda}}(t) \in \partial F$. ■

Lemma 4: $\boldsymbol{\lambda}(t)$ converges surely to a unique globally asymptotically stable equilibrium point.

Proof: Let $\tilde{F}$= $-F$. Thus $\tilde{F}$ is the function to be minimized. Consider a continuous Lyapunov function $V(\boldsymbol{\lambda}) = \tilde{F}(\boldsymbol{\lambda}) - \tilde{F}(\boldsymbol{\lambda^*})$. Thus $V(\boldsymbol{\lambda}^*) = 0$, where $\boldsymbol{\lambda}^*$ is the optimal point. Note that $V(\boldsymbol{\lambda}) \geq 0$ and this function is not differentiable. For such non smooth Lyapunov function, the condition for stability in terms of the Dini derivative D^+ can be expressed as

$$\langle \phi, D^+ V(x) \rangle \leq 0, \quad \phi \in -\partial \tilde{F}. \tag{26}$$

But, $D^+ V(\boldsymbol{\lambda}) \in \partial \tilde{F}$. Thus (26) is satisfied. As the minimum of F is unique, $\boldsymbol{\lambda}(t)$ surely converges to optimal point and is stable. ■

The boundedness of the algorithm can be proved by assuming $\lambda_i \in [0, L] \; \forall \, i, L \geq 0$.

APPENDIX II

BOUNDEDNESS OF ITERATES

For proving the boundedness of the iterates $\boldsymbol{\lambda}$, we use a variation of the technique adopted for proving the boundedness of a linear stochastic approximation algorithm in [21].

Consider the following iteration to update λ_i, with $\tilde{h}(\lambda_i) = \frac{h(\boldsymbol{\lambda})}{\lambda_i}$,

$$\lambda_i(n+1) \;=\; \lambda_i(n) + \alpha(n)\left(\tilde{h}_i(\boldsymbol{\lambda})\lambda_i + M_i(n+1)\right).$$

We impose the following assumption for proving the boundedness.

Assumption 1: The channel gain $x(n) \in (0, \infty)$

The assumption is valid for most denisty functions used in modeling the channel.

Assumption 2: $\lambda_i(n) > 0$

This assumption is imposed for the boundedness of the function $\tilde{h}_i(\boldsymbol{\lambda}(n))$. For the linear programming problem considered, the constrained is satisfied at the boundary. This means $\lambda_i^* > 0$, whenever $C_i > 0$.

Assumption 3: $h_i(\boldsymbol{\lambda}) - C_i \neq 0$ for $\boldsymbol{\lambda} \neq \boldsymbol{\lambda}^*$.

We next follow similar approach as in [21]. Important intermediate steps are stated.

We define iterations $\hat{\lambda}_i^j(n)$,

$$\begin{aligned}\hat{\lambda}_i^j(n+1) &= \hat{\lambda}_i^j(n) + \alpha(n)\left(\tilde{h}_i(\boldsymbol{\lambda}(n)\frac{\lambda_i(n)}{\max(1,|\boldsymbol{\lambda}(n_j)|)}\right) + \alpha(n)\frac{M_i(n+1)}{\max\left(1,|\boldsymbol{\lambda}(n_j)|\right)} \qquad (27)\\ &= \hat{\lambda}_i^j(n) + \alpha(n)\left(\tilde{h}_i(\boldsymbol{\lambda}(n))\hat{\lambda}_i^j(n) + \hat{M}_i(n+1)\right).\end{aligned}$$

where, $\hat{\lambda}_i^j(n) = \frac{\lambda_i(n)}{\max\left(1,|\boldsymbol{\lambda}(n_j)|\right)}$, $n \geq n_j$, $n_{j+1} = \min\left\{n > n_j \,\middle|\, \sum_{l=n_j}^{n-1}\alpha(l) > T\right\}$, $n_0 = 0, T > 0$ and $|\quad|$ denotes the max norm.

Iteration (27) has the structure of the basic iteration discussed in [21]. Following the stopping time formulation in [21], using the Assumptions 1, 2, 3 and the boundedness of $\left|\frac{\log\left(1+(\lambda_i-\frac{1}{x_i})^+\right)}{\lambda_i}\right|$, it can proved that,

$$\sup_n |\boldsymbol{\lambda}(n)| < \infty, \text{ w.p. } 1, \qquad (28)$$

which means the iterates are bounded.

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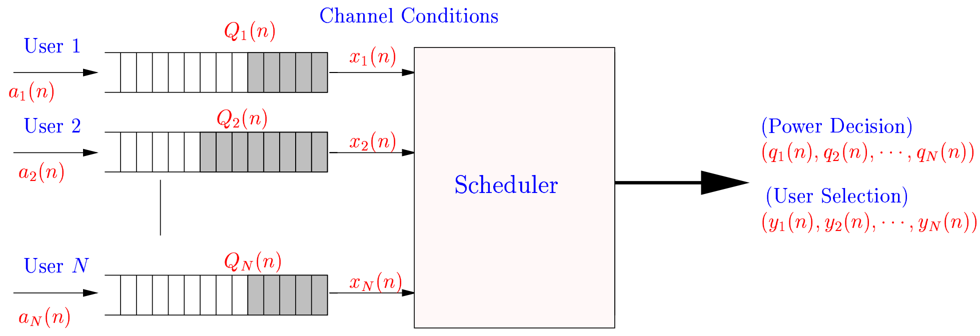

Fig. 1. Single hop system model

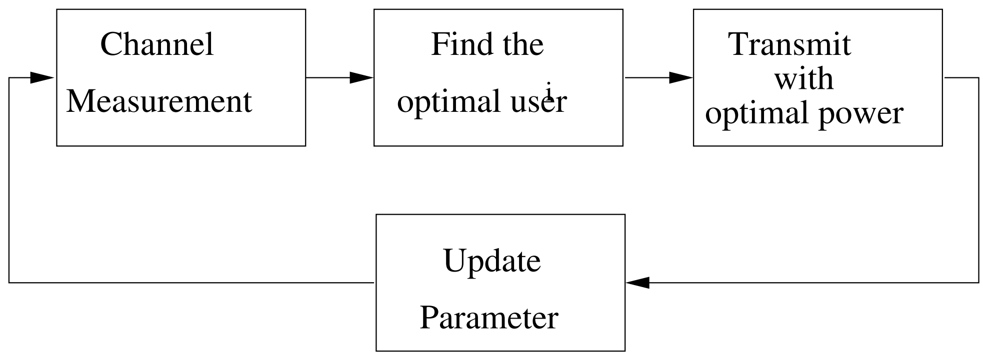

Fig. 2. Block diagram for on-line policy

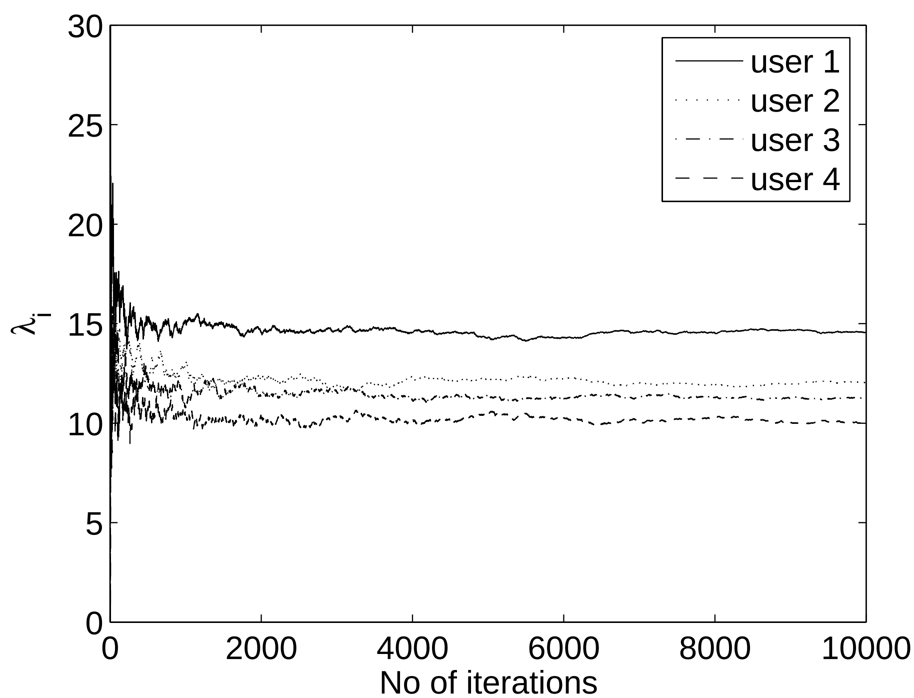

Fig. 3. Convergence for i.i.d. channel

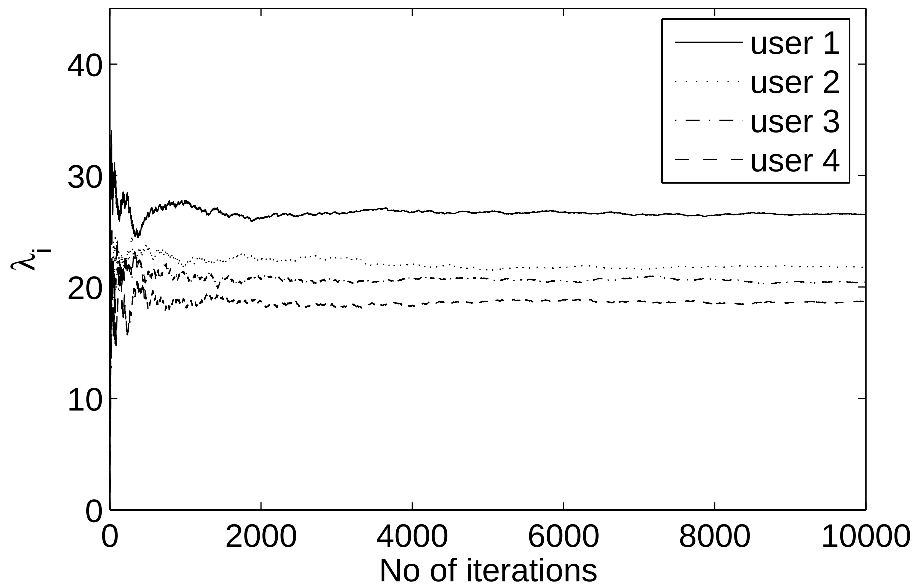

Fig. 4. Convergence for Markovian channel

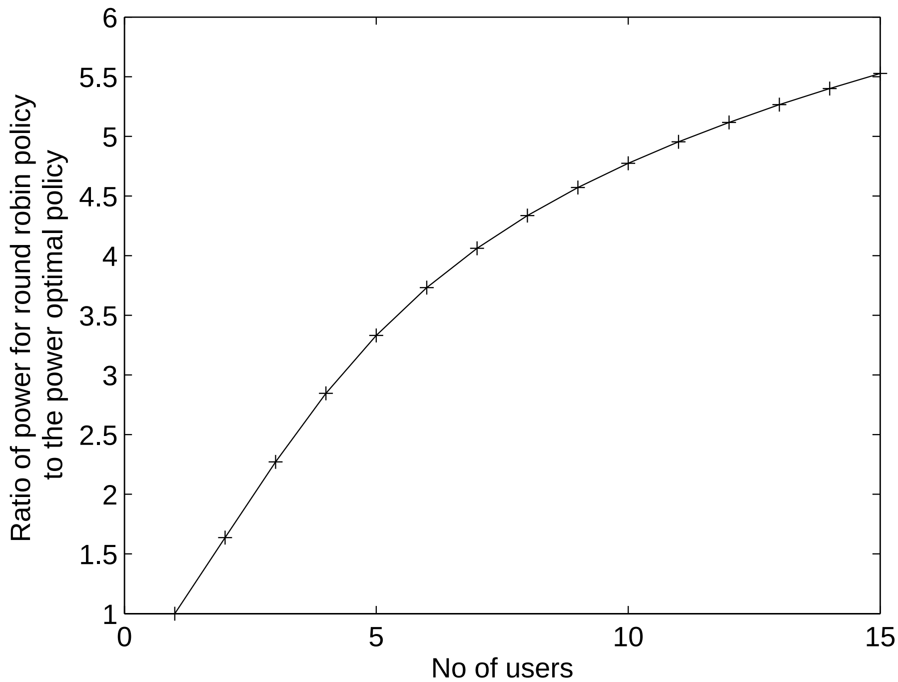

Fig. 5. Gain of the optimal policy over round robin policy

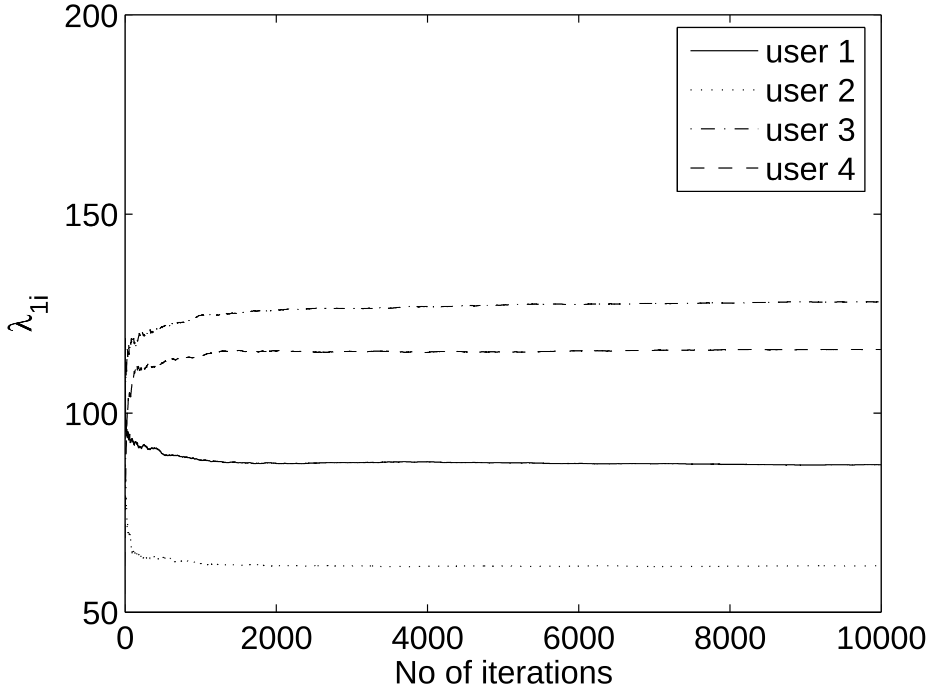

Fig. 6. Trajectory of $\lambda_i(n)$

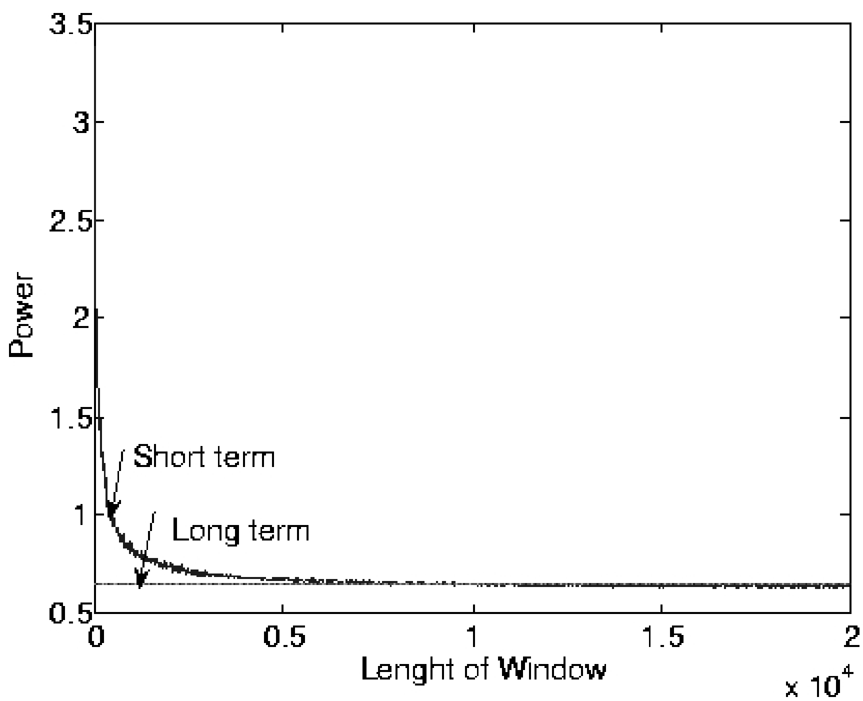

Fig. 7. Power required for the short term fairness compared to long term fairness

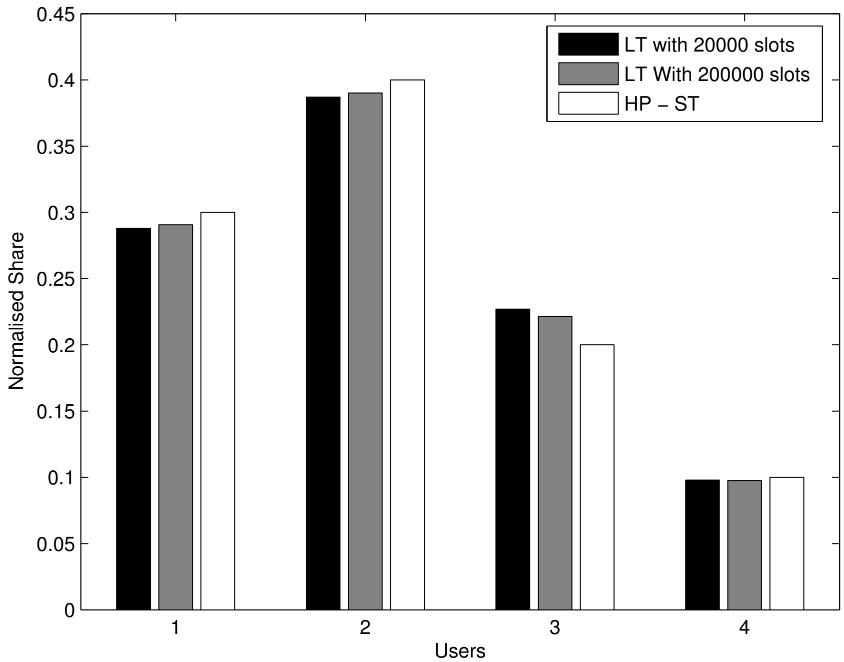

Fig. 8. Fair achieved by short term fair scheduler