Blind Beamforming for Cyclostationary Signals

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Course Project Report

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References:

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1 Project Aim

In this project we studied and implemented *three blind* adaptive beamforming techniques for cyclostationary signals.

Array processing techniques like MVDR, MPDR and others are rather conventional array signal processing techniques. These don't make use of the additional structure present in signals in application specific situations. Signals such as those encountered in communications have been known to display additional structure. One such statistical property is ***cyclostationarity.*** This knowledge can potentially be used to develop better signal processing strategies as shown in this report.

Relevance of the Project to the coursework: *Several beamforming techniques like the MVDR, MPDR, LCMV among many others were introduced in the course. The beamforming techniques suggested in the project extend these techniques and their applications to a more specialized environment of communication signals. Thus it builds on the theory presented in the lectures and demonstrates an application of array processing in the domain of blind signal processing.*

2 Brief Description of the Project

Three algorithms have been discussed for blind array beamforming: **CAB** (Cyclic Adaptive Beamformer), **C-CAB** (Constrained Cyclic Adaptive Beamformer), and **R-CAB** (Robust Cyclic Adaptive Beamformer). These algorithms achieve signal selectivity by exploiting a unique statistical parameter associated with a cyclostationary signal – the ***cycle*** frequency α.

Every cyclostationary signal has a unique cycle frequency which depends on the carrier frequency, baud rate and the sampling rate. The cyclic (or conjugate cyclic) correlation of such a signal exhibits spectral line components at these cycle (or conjugate cycle) frequencies. On the other hand, stationary noise signals have non-trivial cycle frequencies. The adaptive blind beamforming algorithms are based on the assumption that the cycle frequency of the desired signal is different from the interferer. It is this property that distinguishes signal from the interferer. This assumption is not restricted as the desired signal and the interferer have different features.

The **SCORE** (Spectral Self-Coherence Restoral) is an alternative blind beamforming technique for cyclostationary signals, but it has been shown[3] to suffer from a slow convergence speed and low output signal to noise ratio. In addition the computation complexity of SCORE is high. The new techniques are shown to outperform the SCORE algorithm. (*However, In our study, we have* ***not implemented*** *the SCORE algorithm).*

We have **successfully** implemented the CAB and C-CAB algorithms for the different setups listed below.

1. Carrier Recovery – Recover a desired signal based on difference in carrier frequency (and hence cycle frequency) of user and interferer.
2. DOA estimation – Estimating direction of arrival of a moving source (in presence of interferer)
3. Multipath Signal Recovery – Carrier recovery for signal having multipath component.
4. Multiple Signal Recovery – Carrier recovery for multiple uncorrelated desired signals having the same cycle frequency. (*The results we got for this case are different from that of the paper firstly because we have not been able to simulated uncorrelated signals. Therefore our results cannot be compared with that of the paper, which is for perfectly uncorrelated signals only. Secondly, the paper does not give any results for extracting correlated signals with the same cycle frequency).*

3 Beamforming Techniques

3.1 Conventional Beamforming

It is primarily of two kinds. Both of them suffer from disadvantages.

1. Based on DOA estimation: It is computationally intensive and requires precise array calibration.
2. Based on known training signal: Requires synchronization and sacrifice of bandwidth for training signal.

3.2 Blind Beamforming

1. No reference signal required: Selectivity is achieved using signal specific properties (like cyclic frequency).
2. No advance knowledge of the correlation properties: No knowledge of correlation properties is required. (Signals might be correlated or uncorrelated).
3. No Calibration is necessary: Since DOA is not being estimated, Calibration is not necessary.
4. Selectivity is achieved using knowledge of cycle frequency.

4 Cyclostationarity

$$z(t) = s(t)e^{j2\pi f_c t}$$

$$s(t) = \sum_{k=-\infty}^{+\infty} b(k)g(t-kT)$$

z(t) is a narrow band signal modulated by a carrier at f_c. s(t) is the corresponding base-band signal. b(k) is a random binary sequence (Ex: BPSK modulation) and g(t) is a band-limited pulse shape (Ex: Raised Cosine).

4.1 Cyclotationary Statistics

If b(K) is random, s(t) does not contain first order periodicities. However, $b^2(t) = 1$ (**BPSK**)and therefore $s^2(t)$ (Ignoring contribution from cross terms) is given as

$$s^2(t) = \sum_{k=-\infty}^{+\infty} b^2(k)g^2(t-kT)$$

Hence $s^2(t)$ is effectively periodic with a time period of T. Hence, it contains spectral lines at multiples of baud rate, and more specifically a DC component. $z^2(t)$ hence contains spectral line at +-2fc. And if the signal is sampled at multiple of baud rate, it has spectral lines at $\alpha = (\pm 2fc \pm m f b)$. Thus the cycle frequency of the signal can be controlled by choosing any of the different parameters of carrier, baud and sampling. And associated with these features, different signals have different cycle frequencies [4].

5 Data Model

$$\mathbf{x}(n) = \sum_{k=1}^{K} \mathbf{d}(\theta_k) s_k(n) + \mathbf{i}(n) + \mathbf{v}(n)$$

$s_k(n)$, k= 1,…….,K K narrowband signals from DOA $d(\theta_k)$
i(n) Interferers, **v**(n) white noise
x(n) is Mx1 complex vector, M = array size

Given x(n), input data sequence, we want to recover $s_k(n)$. We estimate $s_k(n)$ as

$$\hat{s}_k(n) = \mathbf{w}_k^H \mathbf{x}(n)$$

where $\mathbf{w_k}$ is the weighting vector(for the k^{th} user) chosen according to several desired optimization criteria, $s_k(n)$ is the estimate of $s_k(n)$.

6 Cyclic Frequency

6.1 Cyclic Correlation (CC)

The cyclic correlation function for a signal s(n) is a 2D function of the shift n_o and the cyclic frequency α and is given as

$$\Phi_{ss}(n_o, \alpha) = \overline{[s(n)s^*(n+n_o)e^{-j2\pi\alpha n}]}_\infty$$

The $[.]_\infty$ time average over infinite observation period. Consider the trivial case when the signal contains a DC component. Then at $n_o = 0$ (no time shift) and α = 0 (DC), the signal has a spectral peak. Thus it has the trivial cycle frequency α = 0.

6.2 Cyclic Conjugate Correlation (CCC)

The cyclic *conjugate* correlation function for a signal s(n) is a 2D function of the shift n_o and the cyclic *conjugate* frequency α and is given as

$$\Phi_{ss^*}(n_o, \alpha) = \overline{[s(n)s(n+n_o)e^{-j2\pi\alpha n}]}_\infty$$

A signal is described as *cyclostationary* if its CC or CCC function is non-zero at n_o and frequency shift α and α is said to be the cycle frequency or cycle conjugate frequency respectively. For α = 0, it reduces to a trivial auto-correlation of the process s(n).

$$\hat{R}_{xu} = \begin{Bmatrix} \Phi_{xx}(n_o, \alpha) & \text{if } u(n) = x^*(n+n_o)e^{j2\pi\alpha n} \\ \Phi_{xx*}(n_o, \alpha) & \text{if } u(n) = x(n+n_o)e^{j2\pi\alpha n} \end{Bmatrix}$$

The blind adaptive beamforming algorithms are based on computing the Beamformer weights that maximizes the CC (or CCC) function at the known cycle frequency of the desired signal. *For our implementation, we used the cyclic conjugate correlation function.*

7 Blind Beamforming Algorithms

7.1 Cyclic Adaptive Beamforming (CAB)

The CAB algorithm maximizes the CCC function for a particular known shift and cycle frequency α of the desired signal. The required cost function is

$$\max_{\mathbf{w},\mathbf{c}} |\Phi_{\hat{s}\hat{v}}(n_o,\alpha)|^2 = \max_{\mathbf{w},\mathbf{c}} |\mathbf{w}^H \hat{\mathbf{R}}_{xu}\mathbf{c}|^2 : \mathbf{w}^H\mathbf{w} = \mathbf{c}^H\mathbf{c} = 1$$

where $\mathbf{v}(n) = \mathbf{c}^H\mathbf{u}(n)$. ($\mathbf{u}(n)$ is time and phase shifted $\mathbf{x}$)
Additional constraints (norm = 1) are imposed to limit the amplitude of **w** and **c**. (vector **c** is a don't care solution. **w** captures the information about the signal direction)

The solutions **w**,**c** to the above optimization problem, denoted by $\mathbf{w}_{CAB}$ and $\mathbf{c}_{CAB}$ are given as the ***left and right singular vectors*** of the matrix $\mathbf{R}_{xu}$ corresponding to the largest singular value [1].

It has been shown in [1] that under the assumption that the desired signal is uncorrelated with the interference at the chosen cycle frequency of the signal, the weight vector $\mathbf{w}_{CAB}$ is a consistent estimate of $d(\theta_k)$.

$$\mathbf{w}_{CAB} \propto \mathbf{d}(\theta): \quad \text{as } N \to \infty$$

Multiple desired signals (same α)...
So far we have dealt with the single user case. When multiple desired users having the same cycle frequency are present, the CAB algorithm can achieve signal selectivity if the angular separation of the signals is larger than the main lobe beamwidth.

CAB does not consider suppression of the interferers. Therefore in the case of strong interferers, performance of CAB may deteriorate.

7.2 Constrained Cyclic Adaptive Beamforming(C-CAB)

C-CAB is basically MPDR with DOA vector $\mathbf{d}(\theta)$ replaced by its consistent estimate $\mathbf{w}_{CAB}$. Weights for the C-CAB are given by

$$\mathbf{w}_{CCAB} = \hat{\mathbf{R}}_{xx}^{-1}\mathbf{w}_{CAB}$$

7.3 Robust Cyclic Adaptive Beamforming(R-CAB)

CAB algorithm is sensitive to perturbation of $\mathbf{R}_{xx}$. Therefore a robust beamforming criterion would be given by

$$\max_{\mathbf{w}} \frac{|\mathbf{w}^H\mathbf{d}|^2}{\mathbf{w}^H\mathbf{R}_I\mathbf{w}} \quad \text{subject to} \quad \frac{|\mathbf{w}^H\mathbf{d}|^2}{\mathbf{w}^H\mathbf{w}} = \delta\ , \quad \mathbf{w}^H\mathbf{d} = 1$$

where $\mathbf{R}_I$ is the autocorrelation of the interferers and δ is a positive number.
The solution of this robust Beamformer has been shown to be

$$\mathbf{w} \propto (\mathbf{R}_I + \gamma\mathbf{I})^{-1}\mathbf{d}$$

where γ is related to δ, but there exists no closed form expression relating these two parameters [3].

8 Fast Adaptive Implementation

The above three cyclic beamforming algorithms require an SVD with complexity of $O(M^3)$ (*where M is the array size*). The fast implementation techniques described in this section can bring down the computational complexity significantly for the case of a single desired signal.

8.1 Fast Computation for CAB, C-CAB

$$\hat{R}_{xu} = \begin{bmatrix} \hat{\sigma}_{11} & \hat{\sigma}_{12} & \cdots & \hat{\sigma}_{1M} \\ \hat{\sigma}_{21} & \hat{\sigma}_{22} & \cdots & \hat{\sigma}_{2M} \\ & \ddots & & \\ \hat{\sigma}_{M1} & \hat{\sigma}_{M2} & \cdots & \hat{\sigma}_{MM} \end{bmatrix}$$

The matrix $\mathbf{R}_{xu}$ is rank one for the single user case. Therefore $\mathbf{w}_{CAB}$, the left singular vector of $\mathbf{R}_{xu}$ can be obtained as

$$\mathbf{w}_{CAB} = \left[\sum_{i=1}^{M} \hat{\sigma}_{1i} \quad \cdots \quad \sum_{i=1}^{M} \hat{\sigma}_{Mi} \right]^T$$

For a given $\mathbf{R}_{xu}$ this fast implementation of $\mathbf{w}_{CAB}$ reduces the order of complexity from $O(M^3)$ to $O(M)$. The CCAB weights can be obtained in term of $\mathbf{w}_{CAB}$ as shown in section 7.2. The order of complexity of $\mathbf{w}_{CCAB}$ can be reduced to $O(M^2)$. Next we need a recursive estimate of the $\mathbf{R}_{xu}$.

8.2 Adaptation of the fast algorithms

$$\mathbf{R}_{xu}(N) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{x}(n)\mathbf{u}^H(n)$$
$$= \frac{N-1}{N} R_{xu}(N-1) + \frac{1}{N} x(N)u^H(N)$$

An estimate of the input correlation matrix $\mathbf{R}_{xu}$ can be obtained by averaging over the outer product between x(n) and u(n).

$$w_{CAB}(N) = \frac{N-1}{N} w_{CAB}(N-1) + \frac{1}{N}\sum_{i=1}^{M} u_i^*(N)x(N)$$

The recursive expressions for the $\mathbf{R}_{xu}$ and the $\mathbf{w}_{CAB}$ are given above. The excursive expression for $\mathbf{w}_{CCAB}$ can be obtained as

$$w_{CCAB}(N) = R_{xu}^{-1}(N) w_{CAB}(N)$$

where R_{xu}^{-1} can be obtained from the $\mathbf{R}_{xu}(N)$ by using the matrix inversion lemma.

9 Simulations

The performance of the blind beamforming algorithms were examined by carrying out the simulation as suggested in [1]. We have obtained the performance results for CAB and CCAB for four different simulation environments. We have used the standard uniform linear array in all our setups.

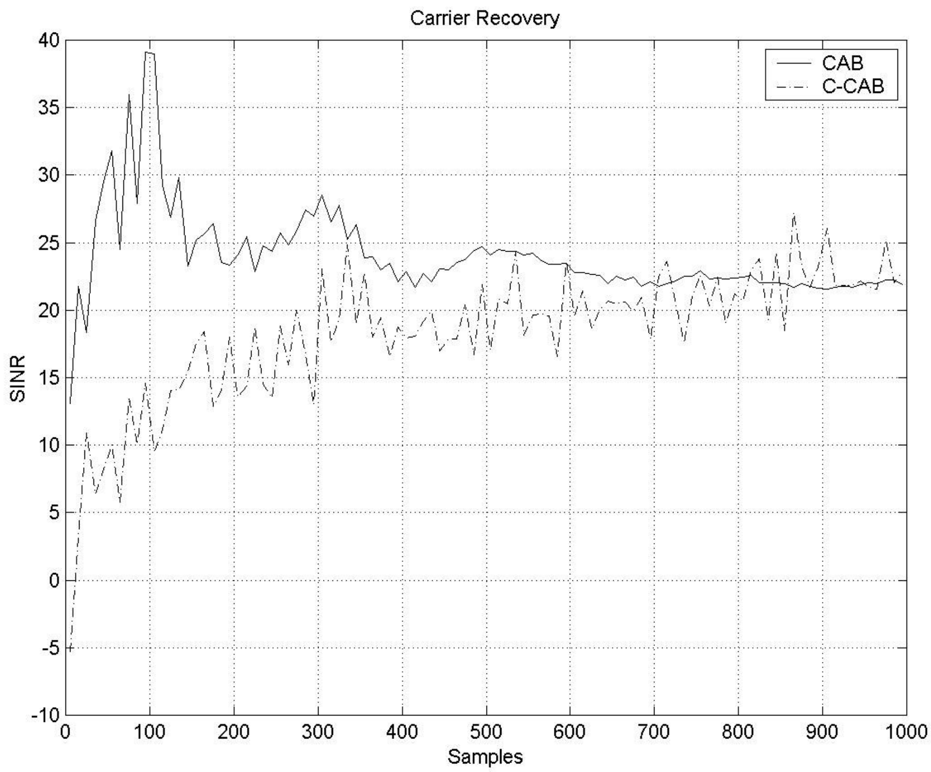
9.1 Experiment 1: Carrier recovery

- In this experiment we have two BPSK signal with 100% cosine roll off arriving at the array. One is the desired signal and the other is the interference.
- The desired signal and the interferer have the same baud rate of 5Kbps. The baud rate is 1/5 times the sampling rate.
- The carrier frequency is 5MHz.
- The signal DOA is 40^o , interferer DOA is 120^o
- The background noise is white
- The CCC function is used and $\alpha = 0$
- The array size M = 6
- The signal and the interferer have a carrier offset of 0.00314

Results

- The figure 1 shows the plot of output SINR(signal to interference plus noise ratio) vs the number of the input data samples.
- We have successfully achieved the signal selectivity using CAB and CCAB.
- These results match with the results given in the paper (refer figures 1 (a), (b) of [1]). However the graphs that we have obtained would not be *identical* to that in the paper as we have used different SNR values.
- The performance of the CAB is better than the CCAB algorithm. This is because the CAB performs better when the signal is stronger than the interferer. Also in section 7.1 we have assumed that the weight $\mathbf{w}_{CAB}$ is the consistent estimate of the DOA vector $\mathbf{d}(\theta)$. In practice due to finite number of samples the $\mathbf{w}_{CAB}$ would not point along $\mathbf{d}(\theta)$ but there would be an offset. This mismatch in the $\mathbf{w}_{CAB}$ and estimation error in $\mathbf{R}_{xu}$ would further deteriorate the performance of CCAB as it is not a robust algorithm.

<u>Figure 1</u>



9.2 Experiment 2: DOA Estimation

- In this experiment we carried out the DOA estimation of a moving source.
- We have a source moving at a speed of 100mph and at a distance of 100m from the array and we wish to estimate its DOA. There is an interferer at 30^o.
- The source DOA range from 40º - 130º
- The signal SNR = 8dB and interference SNR = 4dB.
- The source and the signal have the same carrier frequency but different baud rate (relative baud rate of 1/9)
- The array size M = 16
- The sampling rate is 150Ksamples/s
- We calculate the Beamformer weight vectors and update it every 0.1s using most recent 60 symbols (300 samples).

Results

- Figure 2 shows the plot of the beam pattern vs. the DOA of the moving source.
- Figure 3 shows the plot of the estimated DOA and true DOA versus the number of updates.
- We have used the CAB algorithm to track the moving source. We see from figure 2 that the beam pattern is able to correctly track the moving source. (Notice the peak of the beam pattern shifts as the DOA increases)
- The two curves in figure 3 almost coincides which implies that the CAB algorithm is able to track the source completely i.e. the estimated DOA is very close to the true DOA value.
- These two plots establish the fast convergence speed of the CAB algorithm.

- The results match with the results given in the paper(refer fig. 4 (a),(b) of [1] for comparison)
- In this experiment the blind beamformer is able to suppress the interferer due to the different baud rate which results in different cycle frequency for the signal and the interferer.

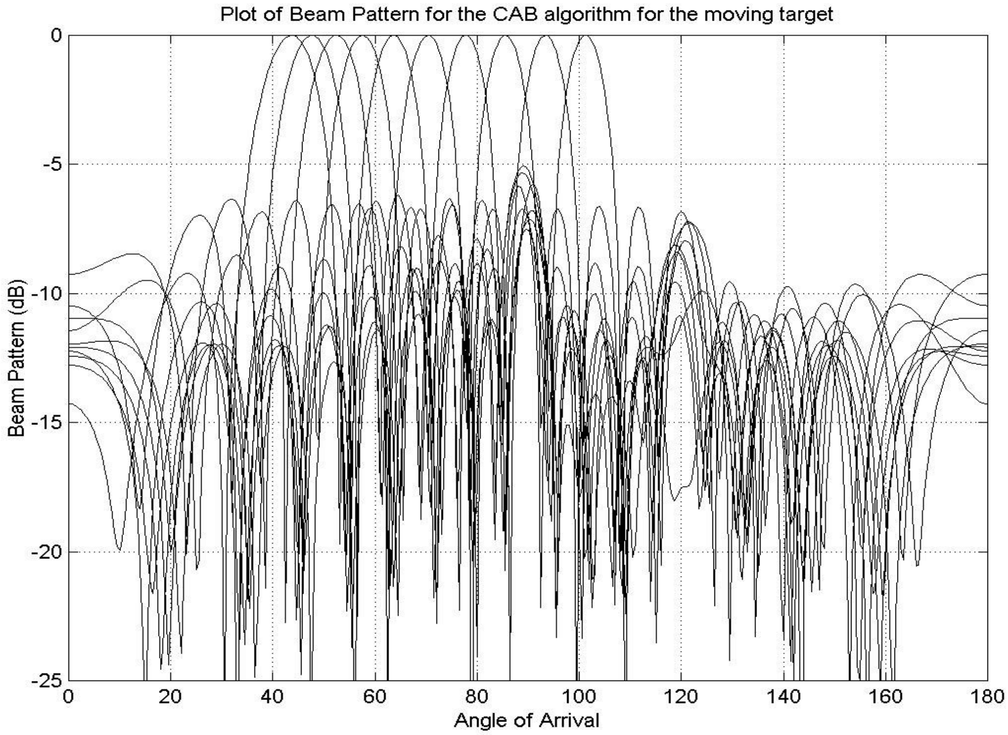
9.3 Experiment 3: Carrier recovery for the Multipath signals

- In this experiment we carried out the carrier recovery for the multipath signals.
- We have a signal and its multipath component impinging at the array at different angles. The signal is at 30ºand the multipath component is at 40º with SNR of 15dB and 12dB respectively.
- The array size M = 10
- There is an interferer at 120º with SNR = 1dB and carrier offset with respect to the signal.

Results

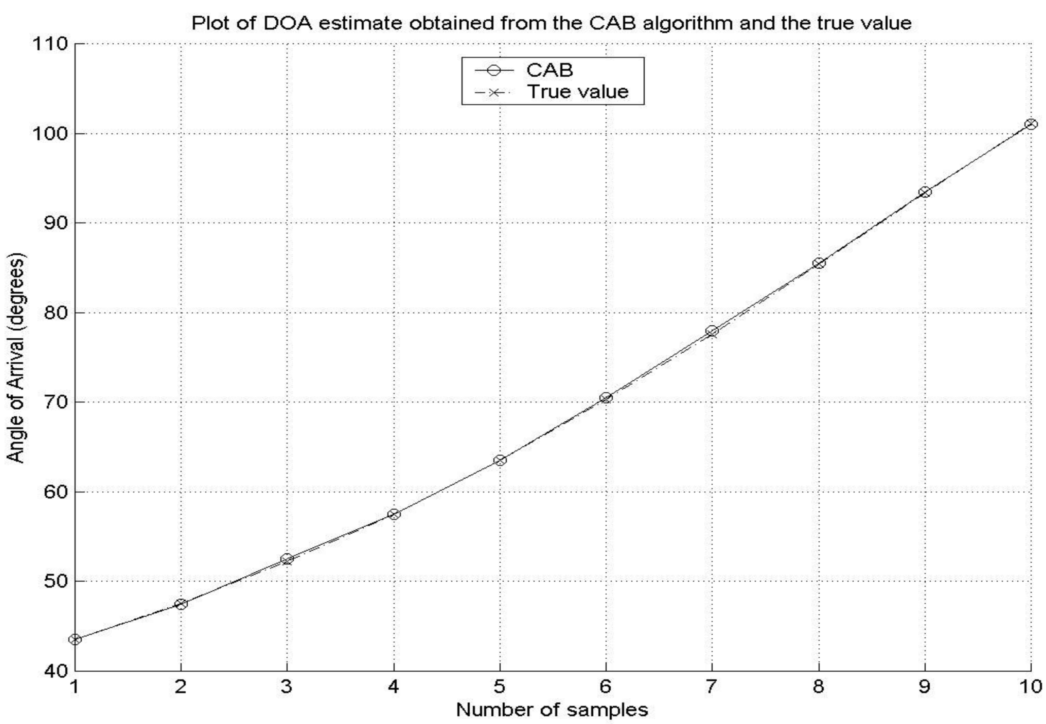
- Figure 4 shows the plot of the output SINR vs the number of input data samples.
- The CAB and the CCAB both successfully recovered the signal.
- The results match with the results given in the paper (refer fig. 3 of [1] for comparison). However the graphs that we have obtained would not be *identical* to that in the paper as we have used different SNR values.

Figure 2



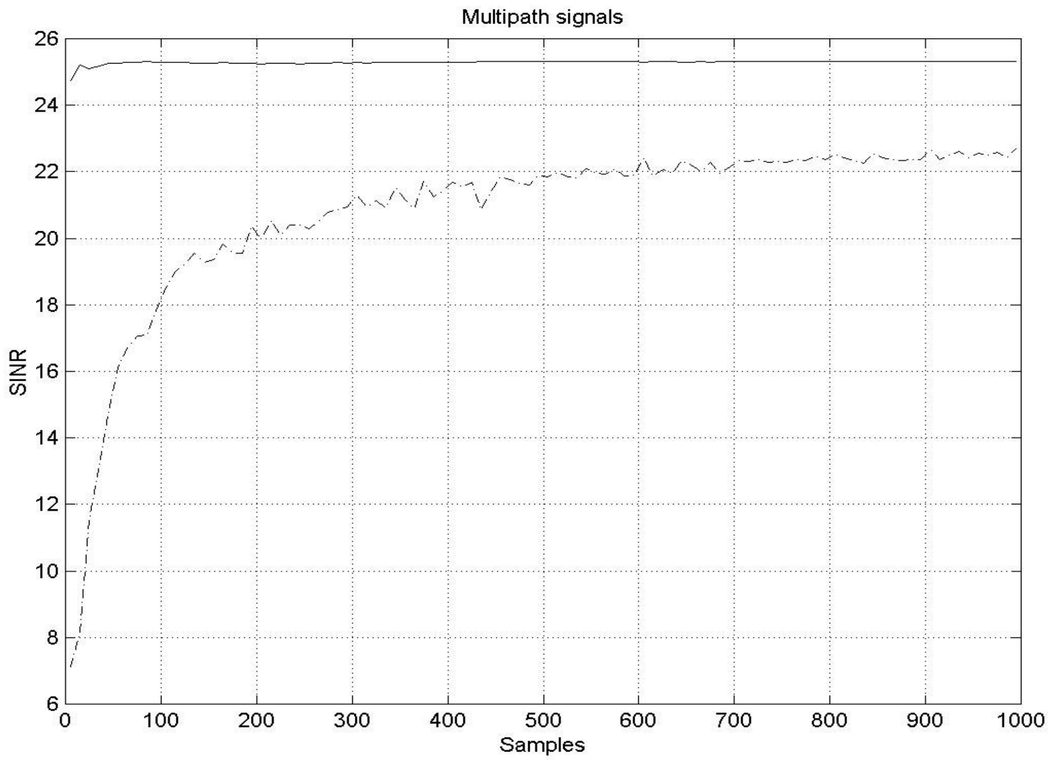
Plot of Beam Pattern for the CAB algorithm for the moving target
Beam Pattern (dB)
Angle of Arrival
0
-5
-10
-15
-20
-25
0
20
40
60
80
100
120
140
160
180

Figure 3



Plot of DOA estimate obtained from the CAB algorithm and the true value
CAB
True value
Angle of Arrival (degrees)
Number of samples
110
100
90
80
70
60
50
40
1
2
3
4
5
6
7
8
9
10

Figure 4



9.4 Experiment 4: Carrier recovery for the Multiple desired signals

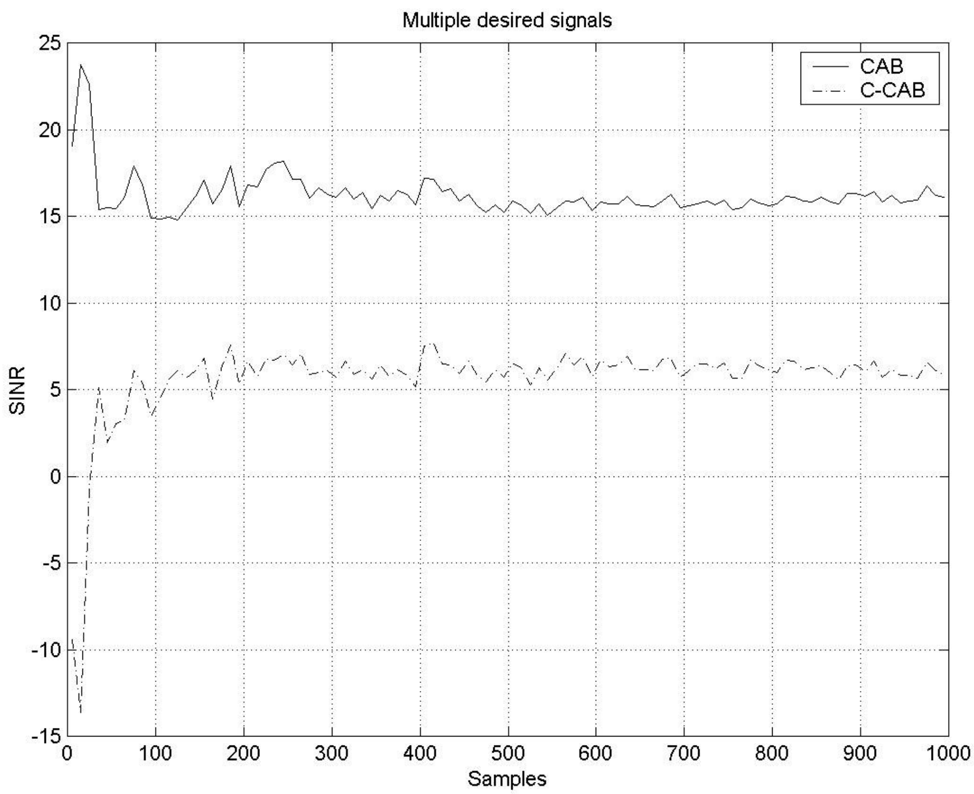
- In this experiment we carried out the carrier recovery for the multiple desired signals.
- We have two desired signals with *the same carrier frequency and the same baud rate*. This means that the two signals have the same cycle frequency. The desired signal is recovered back due to the orthogonal DOA with respect to that of the other signal.
- The DOA for the desired signal is 130º and SNR 15dB. The other signal is at 60º with SNR 9dB
- The interferer is at 10º with strength 1dB. The interferer has a carrier offset and therefore different cycle frequency from that of the signals.
- The array size M = 15

Results

- Figure 5 shows the plot of the output SINR vs. the number of input data samples.
- *The CAB and the CCAB has relatively low output SINR when compared to the results in [1] (refer fig. 2(a) of [1] for comparison).*
- ***This mismatch in the results is due to following***
    1. The simulation ***setup in the paper*** assumed that the two signals are ***uncorrelated*** to start with. We were not able to obtain completely uncorrelated signals. The signals in our simulations had good amount of correlation between them.

2. So we had two correlated signals at same carrier frequency but at orthogonal DOA. Ideally according to the theory we should still be able to recover the signal back. But we did not get good results. *Neither does the paper show any results for the signal recovery for correlated signals with same cycle frequency.*
3. Therefore we cannot compare our results with that of figure 2(a) in the paper [1].

Figure 5



10 Conclusions

- Achieved blind beamforming exploiting the cyclostationarity property of the communication signal.
- Using structure of the signals efficient signal processing techniques can be developed.