Power Allocation over Slow Fading Channels with QAM Inputs

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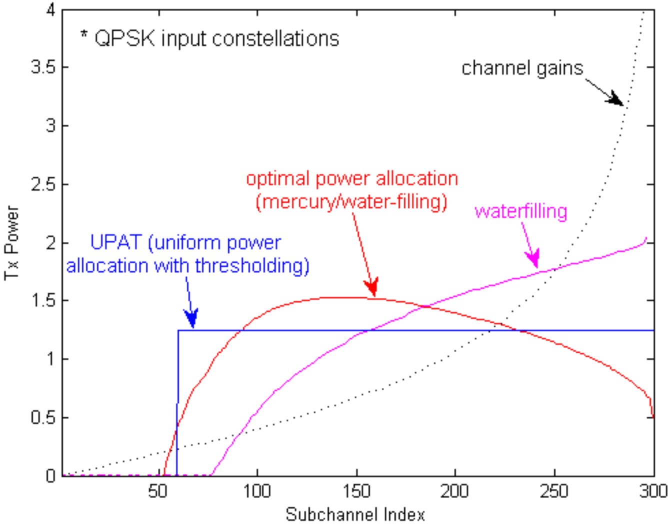
Power Allocation to Maximize the Mutual Information of Parallel Gaussian Channels with QAM Inputs



Outline

I. **Motivation / Summary**
II. **System Model and Performance Metric**
III. **Problem Formulation**
IV. **Power Allocation Schemes**
   - Optimal Power Allocation (Mercury/water-filling)
   - Waterfilling
   - Uniform Power Allocation with Thresholding (UPAT)
V. **Power Allocation Examples**
VI. **Performance Results: Outage Probability**
VII. **Conclusion**

Appendix: Summary of a relevant paper in preparation
"Uniform Power Allocation with Thresholding for Rayleigh Fading and QAM Inputs"

Power Allocation to Maximize the Mutual Information

- **Why mutual information?**
  - Channel capacity is characterized by the maximum mutual information between the input and the output of the channel.
- **Mutual information of parallel Gaussian channels**
  - Parallel Gaussian channels: OFDM, SVD-MIMO, etc.
  - Under an average power constraint, maximized by Gaussian inputs along with the waterfilling power allocation.
- Gaussian inputs can never be realized in practice.
- Rather, the inputs must be drawn from finite discrete constellations such as PSK, PAM, and QAM.
- For these practical discrete input constellations, mercury/water-filling (MWF) is optimal.

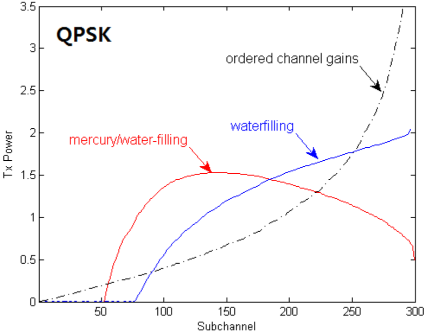
Waterfilling and Mercury/water-filling

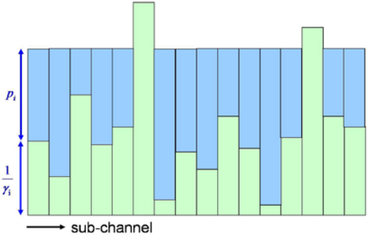
- **Waterfilling**

$$p_i^{wf} = \begin{cases} \frac{1}{\lambda_{wf}} - \frac{1}{\gamma_i}, & \gamma_i > \lambda_{wf} \\ 0, & \gamma_i \le \lambda_{wf} \end{cases}$$

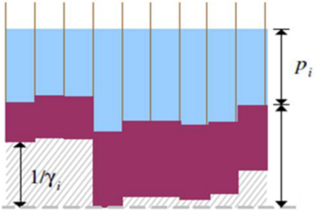
- **Mercury/water-filling**

$$p_i^{mwf} = \begin{cases} \frac{1}{\gamma_i}\text{MMSE}^{-1}\left(\frac{\lambda_{mwf}}{\gamma_i}\right), & \gamma_i > \lambda_{mwf} \\ 0, & \gamma_i \le \lambda_{mwf} \end{cases}$$





(a) waterfilling



(b) mercury/water-filling

Why Study UPAT with QAM Inputs?

- **Concerns on Mercury/water-filling**
  - Feedback overhead
    - Rx $\rightarrow$ Tx: channel gains of each subchannel
    - Tx $\rightarrow$ Rx: power levels of each subchannel
  - Implementation complexity
    - Inverse MMSE functions are involved in MWF

- Much effort has been made to develop simple power allocation schemes

- In particular, UPAT has received much attention thanks to:
  - Remarkably relaxed overhead requirements
  - Simplification of transmitter and receiver design

- However, study on UPAT has focused on the Gaussian input over fast fading channels (ergodic performance)
  - **Insight into practical system design is limited**

Main Results

- Consider
  - SISO Point-to-point communication
  - Slow fading channels (Rayleigh) and the outage probability
  - M-QAM inputs
- Compare three power allocation schemes:
  - Optimal power allocation (Mercury/water-filling)
  - Waterfilling
  - UPAT
- It will be shown that
  - As long as the constellation size M is sufficiently large,
  - UPAT $\approx$ waterfilling $\approx$ Mercury/water-filling.

Outline

I. **Motivation / Summary**

II. **System Model and Performance Metric (Outage Probability)**

III. **Problem Formulation: minimize the outage probability**

IV. **Power Allocation Schemes**

- Optimal Power Allocation (Mercury/water-filling)
- Waterfilling
- Uniform Power Allocation with Thresholding (UPAT)

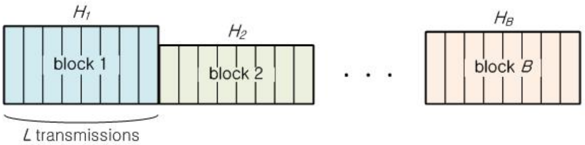
V. **Power Allocation Examples**

VI. **Outage Probability Results**

VII. **Conclusion**

Appendix: Summary of a relevant paper in preparation "Uniform Power Allocation with Thresholding for Rayleigh Fading and QAM Inputs"

System Model: Power Allocation over a Block Fading Ch.



A codeword spans B blocks of L transmissions.
(L: arbitrarily large)

$$\mathbf{Y}_i = H_i \sqrt{p_i(\gamma; M)P}\, \mathbf{S}_i + \mathbf{Z}_i, \quad i = 1, 2, \ldots, B$$

- $\mathbf{Y}_i$: channel output vector in block i
- $\mathbf{Z}_i \sim \mathcal{N}_{\mathbb{C}}(0, \mathrm{I})$
- $\mathbf{S}_i$: the standard M-QAM symbols (unit average power)
- P: average power constraint
- H_i: random channel gain $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$, constant during the block, i.i.d. across the blocks, known to Tx and Rx
- $\gamma_i \triangleq P|H_i|^2$: SNR before power adaptation, $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_B)$
- $p_i(\boldsymbol{\gamma}; M) \geq 0$: normalized Tx power $(\frac{1}{B}\sum_{i=1}^{B} p_i(\boldsymbol{\gamma}; M) \leq 1)$
- $p_i(\boldsymbol{\gamma}; M)\gamma_i$: SNR after power adaptation or instantaneous SNR
- $\mathbf{p}(\boldsymbol{\gamma}; M) = (p_1(\boldsymbol{\gamma}; M), \ldots, p_B(\boldsymbol{\gamma}; M))$: power allocation scheme

Performance Metric: Outage Probability

- **Instantaneous mutual information**

$$I_B(M,\gamma,\mathbf{p}(\gamma;M)) \triangleq \frac{1}{B}\sum_{i=1}^{B} I_{AW}(p_i(\gamma;M)\gamma_i;M)$$

where $I_{AW}(\rho;M)$ is the MI of the AWGN channel with M-QAM inputs at SNR ρ.

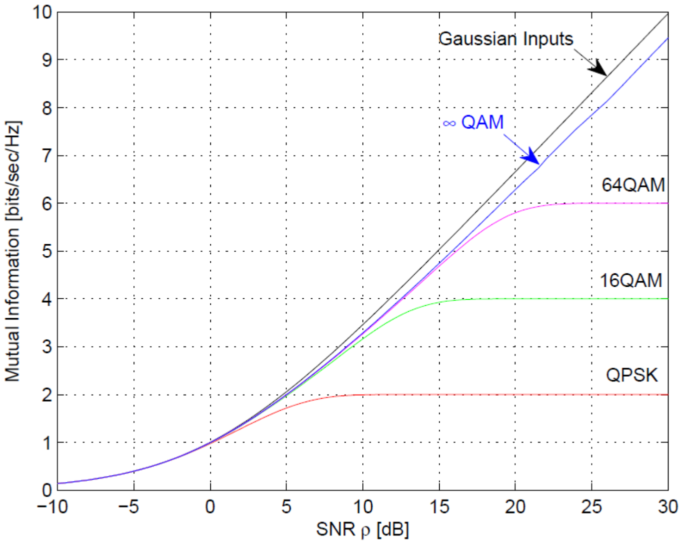
$$I_{AW}(\rho;M) = \log_2 M - \frac{1}{M}\sum_{s\in\mathcal{S}_M} \mathsf{E}_Z\left[\log_2\left(\sum_{s'\in\mathcal{S}_M} e^{-|\sqrt{\rho}(s-s')+Z|^2+|Z|^2}\right)\right]$$

- **Outage probability**

$$P_{out}(B,M,P,R,\mathbf{p}(\gamma;M)) \triangleq \mathsf{P}(I_B(M,\gamma,\mathbf{p}(\gamma;M)) < R)$$

where R is the fixed target transmission rate.

$I_{AW}(\rho; M)$: MI of the AWGN Channel with QAM Inputs



Power Allocation Problem

- **Outage probability minimization problem**

$$\begin{aligned} \text{minimize} \quad & P_{out}(B, M, P, R, \mathbf{p}(\gamma; M)) \\ \text{subject to} \quad & \frac{1}{B}\sum_{i=1}^{B} p_i(\gamma; M) \leq 1 \\ & p_i(\gamma; M) \geq 0, \quad \forall i. \end{aligned}$$

- **Equivalent problem** (under the same power constraint)

$$\arg\max_{\mathbf{p}(\gamma; M)} I_B(M, \gamma, \mathbf{p}(\gamma; M))$$

Outline

	Mercury/water-filling	Water-filling	The optimal UPAT
Problem	$\max_{\mathbf{p}(\gamma;M)} I_B(M, \mathbf{p}(\gamma;M), \gamma)$ s.t. power constraint	$\max_{\mathbf{p}(\gamma;M)} \sum_i \log\left(1 + p_i(\gamma;M)\gamma_i\right)$ s.t. power constraint	$\max_{0 \le n < B} \sum_{i=n+1}^{B} I_{AW}\left(\gamma_i^o \frac{B}{B-n}; M\right)$
Solution $p_i(\gamma;M)$	$\frac{1}{\gamma_i}\mathrm{MMSE}_M^{-1}\left(\frac{\lambda_m}{\gamma_i}\right), \quad \gamma_i > \lambda_m$ $0, \quad \gamma_i \le \lambda_m$	$\frac{1}{\lambda_w} - \frac{1}{\gamma_i}, \quad \gamma_i > \lambda_w$ $0, \quad \gamma_i \le \lambda_w$	$\frac{B}{B-N_{upat}}, \quad \gamma_i > \gamma_{N_{upat}}^o$ $0, \quad \gamma_i \le \gamma_{N_{upat}}^o$

Pout

Mercury/water-filling:

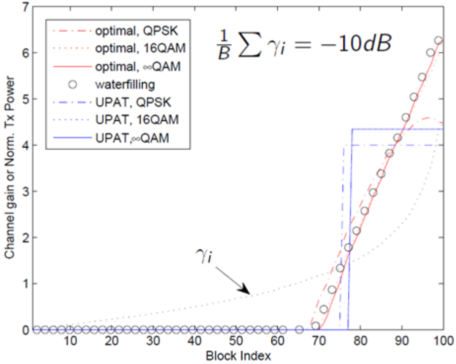
$$\mathrm{P}\left\{\frac{1}{B}\sum_{i:\gamma_i \ge \lambda_m} I_{AW}\left(\mathrm{MMSE}_M^{-1}\left(\frac{\lambda_m}{\gamma_i}\right); M\right) < R\right\}$$

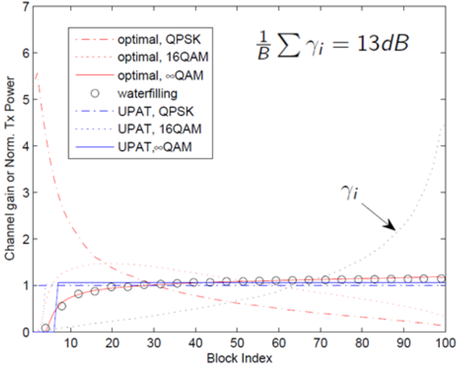
Water-filling:

$$\mathrm{P}\left\{\frac{1}{B}\sum_{i:\gamma_i \ge \lambda_w} I_{AW}\left(\frac{\gamma_i}{\lambda_w} - 1; M\right) < R\right\}$$

The optimal UPAT:

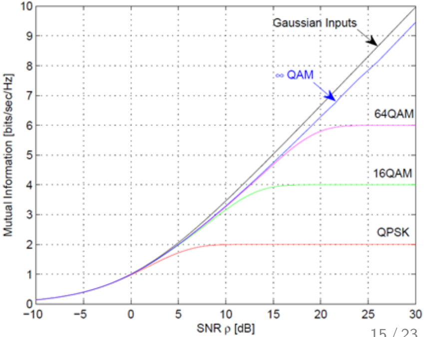
$$\mathrm{P}\left\{\frac{1}{B}\sum_{i=N_{upat}+1}^{B} I_{AW}\left(\gamma_i^o \frac{B}{B-N_{upat}}; M\right) < R\right\}$$

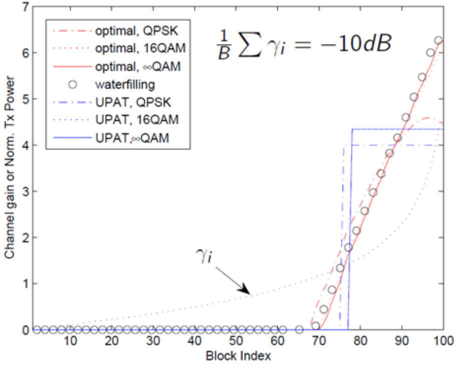


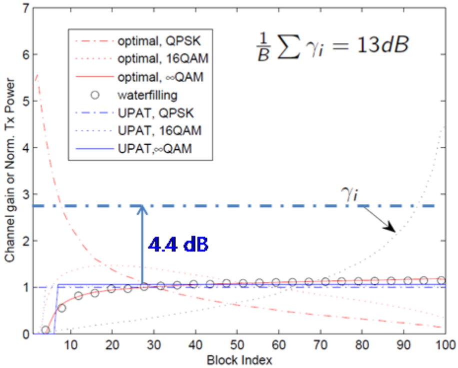


Case	$I_B(MWF)$	$I_B(UPAT)$	Loss	ΔP
−10dB, QPSK	0.2314	0.2288	1.2 %	0.07 dB
−10dB, 16QAM	0.2385	0.2341	1.8 %	0.12 dB
−10dB, ∞QAM	0.2394	0.2349	1.9 %	0.12 dB
13dB, QPSK	1.9499	1.8630	4.5 %	4.4 dB
13dB, 16QAM	3.1917	3.1199	2.3 %	0.5 dB
13dB, ∞QAM	3.5950	3.5871	0.2 %	0.03 dB

- ΔP is large when (γ, M) is such that $I_{AW}(\gamma_i; M) \approx \log_2 M$ for many i's.







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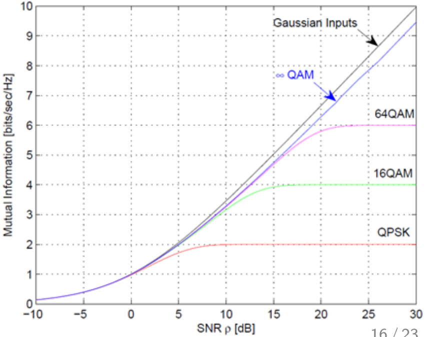

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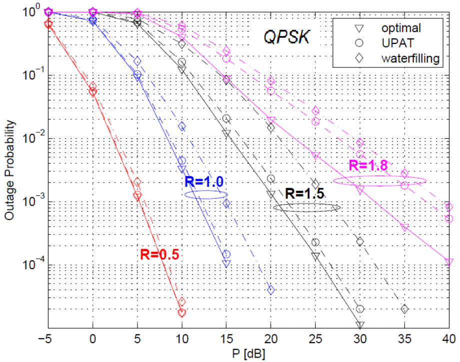
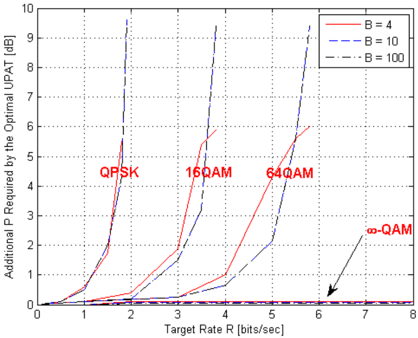
Suboptimality of the Optimal UPAT and Waterfilling

- If $R \ll log_2 M$, the optimal UPAT and waterfilling perform near MWF.
- If $R \approx log_2 M$, the performance loss is significant, especially for waterfilling.

Asymptotic Results: as $R \to 0$,

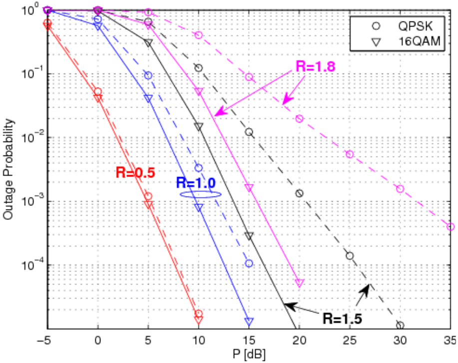
$$P_{out}(UPAT) = P_{out}(WF) = P_{out}(MWF).$$



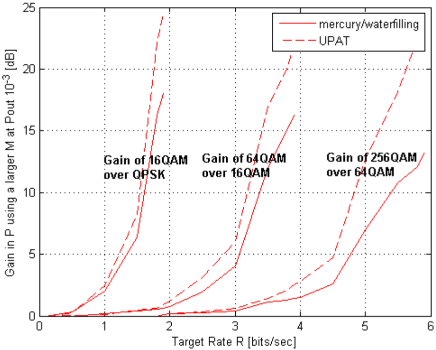


"$R \approx log_2 M$" Should Be Avoided

- **All the schemes perform so poorly** in the regime where $R \approx log_2 M$.
- One-step larger M could significantly improve the performance, leading to the condition $R \ll log_2 M$.



(a) QPSK vs. 16QAM (MWF, $B = 4$)



(b) Gain in P of larger M at $P_{out} = 10^{-3}$ ($B = 4$)

Actual Performance of the Optimal UPAT and Waterfilling

As long as the constellation size M is properly chosen,

The optimal UPAT as well as waterfilling perform near optimal !!!

(Note: The same conclusion holds for fast fading channels where the ergodic mutual information is considered.)

Concluding Remarks

- **UPAT is an attractive power allocation policy in practice.**
- **In many communication problems, e.g., power allocation, multi-user scheduling, MIMO techniques, solutions for optimizing $f(\log(1 + SNR))$ are significant in practice with sufficiently large constellation sizes.**

Appendix: Summary of a Relevant Paper

"Uniform Power Allocation with Thresholding for Rayleigh Fading and QAM Inputs"

- Hwan-Joon(Eddy) Kwon, Young-Han Kim, and Bhaskar D. Rao
- to be submitted to IEEE Transactions on Wireless Communications.

Summary of the Main Results

1) **Analyze the suboptimality of the optimal UPAT**
   - The optimal UPAT performs near MWF as long as the constellation size M is appropriately chosen.
2) **Propose a constellation size selection rule**
   - Provides a good compromise between performance and complexity.
   - With the rule, the optimal UPAT performs near MWF.
3) **Analyze the amount of gain of the optimal UPAT over uniform power allocation**
   - Significant when the target rate R is low and the number of fading DOFs is large.
4) **Propose a simple algorithm to set the threshold for UPAT**
   - Significantly reduces the computational complexity with minimal performance loss.
5) **Extend to Fast Fading Channels**
   - Show that the same conclusion holds.