# D2D Resource Sharing and Beamforming

#### PhuongBang Nguyen

Department of Electrical Engineering University of San Diego, California

Fall 2012

PhuongBang Nguyen (UCSD)





2 Single Antenna Scenario (SISO)





#### Outline



2 Single Antenna Scenario (SISO)

3 Multiple Antenna Scenario (MIMO)

#### 4 References

# **Project Outline**

- This presentation discusses the **resource optimization problem** in a Device-to-Device communications network.
- The problem is examined under several different settings:
  - Single antenna, single carrier (Single-carrier SISO)
  - Single antenna, multiple carriers (Multi-carrier SISO)
  - Multiple antennas, single carrier (Single-carrier MIMO)
- Only brief summaries are presented for the SISO cases.
- The MIMO case is discussed in detail in three sub-topics:
  - Orthogonal Beamforming
  - Zero-forcing Beamforming
  - Tunable Beamforming

# What is Device-to-Device (D2D) Communication?



- User Equipments (UE's) communicate directly with each other.
- D2D connections remain under the control of the base station.

# D2D Link Budget I

• Assume 10 MHz bandwidth with the receiver operating at 290K.

а	Max. TX power (dBm)	24.0	
b	TX antenna gain (dBi)	0.0	
С	Body loss (dB)	0.0	
d	EIRP (dBm)	24.0	= a + b + c
е	RX UE noise figure (dB)	7.0	
f	Thermal noise (dBm)	-104.5	= k * T * B
g	Receiver noise floor (dBm)	-97.5	= e + f
h	SINR (dB)	-10.0	
i	Receiver sensitivity (dBm)	-107.5	= g + h
j	Interference margin (dB)	3.0	
k	Control channel overhead (dB)	1.0	
Ι	RX antenna gain (dBi)	0.0	
m	Body loss (dB)	0.0	
	Maximum path loss (dB)	127.5	= d - i - j - k + l - m

#### D2D Link Budget II

• Using simple a path loss model (Okumura-Hata model)

$$G_{avg} = C - 10\alpha \log_{10} r \tag{1}$$

Where *C* is a correction factor and  $\alpha \approx 3-5$ .

• Using C = -15 dB [1], corresponding to rural areas, and  $\alpha = 5$  for lots of loss due to the fact both UE's are very close the the ground, the maximum range between devices can be computed to be

$$r = 10^{\frac{C - G_{avg}}{10\alpha}} = 10^{\frac{-15 + 127.5}{10 \times 5}} = 10^{2.25} = 178m$$
 (2)

• The D2D operating range is several hundred meters, depending on the environment and handset capabilities.

#### Multiple D2D Links



 Due to the short range of D2D communications, it is possible to have multiple D2D links sharing a common resource with little interference.

# Benefits and Challenges of D2D Communications

#### Benefits

- For the UE's:
  - Better throughput
  - Lower power
  - \* Shorter delay
  - ★ Transparent mode switching
- For the system:
  - ★ Less relay load for the base stations
  - ★ Better channel resource reuse
- For the service provider:
  - Easier to plan access, investment and interference coordination in a licensed band.
  - \* Resource can still be assigned to D2D in a dense network.

#### Challenges

- Peer Discovery
- Mode Selection
- Interference Management/Coordination

#### **D2D Interference Scenarios**



PhuongBang Nguyen (UCSD)

D2D Beamforming





#### 2 Single Antenna Scenario (SISO)

3 Multiple Antenna Scenario (MIMO)

#### 4 References

PhuongBang Nguyen (UCSD)

#### Single-Antenna, Single-Carrier System

• The objective is to maximize the minimum SINR of the two links,  $\Gamma_c$  and  $\Gamma_d$ , subject to individual power constraints.

$$(P_c^*, P_d^*) = \underset{P_c, P_d}{\operatorname{arg\,max}} \{ \min\{\Gamma_c, \Gamma_d\} \} \text{ s.t. } 0 \le P_c, P_d \le P_{max} \quad (P1)$$
Where  $\Gamma_c = \frac{g_c P_c}{g_{dc} P_d + N_c}, \Gamma_d = \frac{g_d P_d}{g_{cd} P_c + N_d}$ 
(3)

- The optimal solution must satisfy  $P_d = P_{\text{max}}$  or  $P_c = P_{\text{max}}$  [2].
- In either case, problem (P1) is a *quasi concave* problem in  $P_c$  or  $P_d$ .
- The solution can be obtained directly by setting  $\Gamma_c = \Gamma_d$  and solving a simple quadratic equation for the power.

#### Single-Antenna, Multi-Carrier System

• We joint-optimize over the shared set of N sub-carriers.

$$\max_{\mathbf{x},\mathbf{y}} \min \{R_c, R_d\} \text{ s.t. } \mathbf{1}^T \mathbf{x} \le P_c^{tot}, \ \mathbf{1}^T \mathbf{y} \le P_d^{tot}$$
(P2)  
$$R_{c,d} \triangleq \sum_{k=0}^{N-1} \log_2(1 + \text{SINR}_{c,d}^{(k)}), x_k \triangleq P_c^{(k)}, y_k \triangleq P_d^{(k)}$$
(4)

• Rewriting problem (P2) using a *slack* variable *s*, we have

$$\max_{\mathbf{x},\mathbf{y},s} \{s\} \text{ s.t. } \mathbf{1}^{T}\mathbf{x} \le P_{c}^{tot}, \ \mathbf{1}^{T}\mathbf{y} \le P_{d}^{tot}$$
(5)  
$$s \times \prod_{k=0}^{N-1} \frac{g_{dc}^{(k)}y_{k} + N_{c}^{(k)}}{g_{dc}^{(k)}y_{k} + N_{c}^{(k)} + g_{c}^{(k)}x_{k}} \le 1$$
(6)  
$$s \times \prod_{k=0}^{N-1} \frac{g_{cd}^{(k)}x_{k} + N_{d}^{(k)}}{g_{cd}^{(k)}x_{k} + N_{d}^{(k)} + g_{d}^{(k)}y_{k}} \le 1$$
(7)

 This problem can be solved as a *geometric program* by using monomial approximation to the denominators of (6) and (7) [3].

PhuongBang Nguyen (UCSD)

#### Single-Antenna, Multi-Carrier System



PhuongBang Nguyen (UCSD)

#### Outline

# 1 Introduction

2 Single Antenna Scenario (SISO)

#### 3 Multiple Antenna Scenario (MIMO)

#### 4 References

# System Model I



- We consider a downlink celular system [4] with D2D enabled under the following conditions:
  - The base station has N antennas.
  - The mobile devices have M antennas, where M < N.
  - Transmit and receive beamformers are used at all terminals.

#### System Model II

 The signals received at the D2D and cellular receivers are given by

$$y_d = \mathbf{u}_d^H \mathbf{H}_d \mathbf{v}_d \sqrt{P_d} x_d + \mathbf{u}_d^H \mathbf{H}_{cd} \mathbf{v}_c \sqrt{P_c} x_c + N_d$$
(8)

$$y_c = \mathbf{u}_c^H \mathbf{H}_c \mathbf{v}_c \sqrt{P_c} x_c + \mathbf{u}_c^H \mathbf{H}_{dc} \mathbf{v}_d \sqrt{P_d} x_d + N_c$$
(9)

#### Where

- $x_c, x_d$  are scalar transmit signals for the cellular and D2D links
- $\mathbf{v}_c, \mathbf{v}_d$  and  $\mathbf{u}_c, \mathbf{u}_d$  are unit-norm transmit/receive beamformers
- $P_c, P_d$  are transmit powers
- ► **H**<sub>c</sub>, **H**<sub>d</sub> are MIMO channel matrices for the direct paths, **H**<sub>cd</sub>, **H**<sub>dc</sub> are channel matrices for the interference paths
- $N_c \sim \mathcal{N}(0, \sigma_c^2)$  and  $N_d \sim \mathcal{N}(0, \sigma_d^2)$  are Gaussian noises.

#### **Problem Formulation**

• Consider the following joint optimization problem

$$\max_{\mathbf{u}_{c},\mathbf{u}_{d},\mathbf{v}_{c},\mathbf{v}_{d},P_{c},P_{d}} \min \{\Gamma_{c},\Gamma_{d}\}$$
(P3)  
s.t.:  $0 \le P_{c} \le P_{c}^{max}, 0 \le P_{d} \le P_{d}^{max}$   
 $\|\mathbf{u}_{c}\| = 1, \|\mathbf{v}_{c}\| = 1, \|\mathbf{u}_{d}\| = 1, \|\mathbf{v}_{d}\| = 1$ 

#### Where

$$\Gamma_{c} = \frac{|\mathbf{u}_{c}^{H}\mathbf{H}_{c}\mathbf{v}_{c}|^{2}P_{c}}{|\mathbf{u}_{c}^{H}\mathbf{H}_{dc}\mathbf{v}_{d}|^{2}P_{d} + \sigma_{c}^{2}} = \frac{\mathbf{u}_{c}^{H}(P_{c}\Phi_{c})\mathbf{u}_{c}}{\mathbf{u}_{c}^{H}(P_{d}\Phi_{dc} + \sigma_{c}^{2}\mathbf{I})\mathbf{u}_{c}}$$
(10)  

$$\Gamma_{d} = \frac{|\mathbf{u}_{d}^{H}\mathbf{H}_{d}\mathbf{v}_{d}|^{2}P_{d}}{|\mathbf{u}_{d}^{H}\mathbf{H}_{cd}\mathbf{v}_{c}|^{2}P_{c} + \sigma_{d}^{2}} = \frac{\mathbf{u}_{d}^{H}(P_{d}\Phi_{d})\mathbf{u}_{d}}{\mathbf{u}_{d}^{H}(P_{c}\Phi_{cd} + \sigma_{d}^{2}\mathbf{I})\mathbf{u}_{d}}$$
(11)  

$$\Phi_{c} = \mathbf{H}_{c}\mathbf{v}_{c}\mathbf{v}_{c}^{H}\mathbf{H}_{c}^{H}, \ \Phi_{d} = \mathbf{H}_{d}\mathbf{v}_{d}\mathbf{v}_{d}^{H}\mathbf{H}_{d}^{H}$$
  

$$\Phi_{dc} = \mathbf{H}_{dc}\mathbf{v}_{d}\mathbf{v}_{d}^{H}\mathbf{H}_{dc}^{H}, \ \Phi_{cd} = \mathbf{H}_{cd}\mathbf{v}_{c}\mathbf{v}_{c}^{H}\mathbf{H}_{cd}^{H}$$

• Problem (P3) is a non convex optimization problem.

# **D2D Optimization Procedure**

- For the performance/complexity tradeoffs under D2D settings, we consider the following optimization procedure:
  - The D2D link ignores the interference from the cellular link and optimizes its own SNR using *Maximal Ratio Transmission (MRT)* [5].

$$\gamma_d = \max_{\mathbf{u}_d, \mathbf{v}_d, P_d} \Gamma_d = \frac{|\mathbf{u}_d^H \mathbf{H}_d \mathbf{v}_d|^2 P_d}{\sigma_d^2} \text{ subject to: } 0 \le P_d \le P_d^{max} \quad (12)$$

Given  $\mathbf{v}_d$ , and  $P_d$ , the base station solves for  $\mathbf{u}_c$ ,  $\mathbf{v}_c$ ,  $P_c$  by minimizing the interference to the D2D link and maximizing the cellular link SINR  $\Gamma_c$ .

$$\min_{\mathbf{v}_c} \left( \mathbf{v}_c^H \mathbf{H}_{cd}^H \mathbf{u}_d \right) \left( \mathbf{u}_d^H \mathbf{H}_{cd} \mathbf{v}_c \right)$$
subject to  $\|\mathbf{v}_c\| = 1$ 
(13)

- Zero interference can be achieved when
  - $\mathbf{v}_c \perp (\mathbf{H}_{cd}^H \mathbf{u}_d)$ : Orthogonal beamforming.
  - $\mathbf{H}_{cd}\mathbf{v}_{c} = \mathbf{0}$ : Zero-forcing beamforming.

### Orthogonal Beamforming I

- In this case,  $\mathbf{v}_c$  must lie in the *orthogonal complement* space of  $\mathbf{H}_{cd}^H \mathbf{u}_d$ , denoted as  $\mathbf{W}^{\perp}$  with dimension N-1.
- Let  $\mathcal{B} = {\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_{N-1}}$  be an ortho-normal basis of  $\mathbf{W}^{\perp}$ , then  $\mathbf{v}_c$  must be a linear combination of  $\mathcal{B}$ .

$$\mathbf{v}_c = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_{N-1}]\mathbf{x} = \mathbf{K}\mathbf{x}$$
(14)

$$\Phi_c = \mathbf{H}_c \mathbf{v}_c \mathbf{v}_c^H \mathbf{H}_c^H = \mathbf{H}_c \mathbf{K} \mathbf{x} \mathbf{x}^H \mathbf{K}^H \mathbf{H}_c^H$$
(15)

Where 
$$\mathbf{x} = [x_1, x_2, ..., x_{N-1}]^T$$

• We want to maximize the SINR  $\Gamma_c$  for the cellular link:

$$\Gamma_{c}^{max} = \max_{\mathbf{x}, \mathbf{u}_{c}} \Gamma_{c} = \max_{\mathbf{x}} \max_{\mathbf{u}_{c}} \frac{\mathbf{u}_{c}^{H} \left(\mathbf{H}_{c} \mathbf{K} \mathbf{x} \mathbf{x}^{H} \mathbf{K}^{H} \mathbf{H}_{c}^{H}\right) \mathbf{u}_{c}}{\mathbf{u}_{c}^{H} B \mathbf{u}_{c}}$$
(16)  
s.t:  $\mathbf{x}^{H} \mathbf{x} = 1$ 

Where  $B = (P_d/P_c)\Phi_{dc} + (\sigma_c^2/P_c)\mathbf{I}$  and  $\Phi_{dc} = \mathbf{H}_{dc}\mathbf{v}_d\mathbf{v}_d^H\mathbf{H}_{dc}^H$ 

# Orthogonal Beamforming II

• Let  $\mathbf{y} \triangleq B^{\frac{1}{2}}\mathbf{u}_c$ , we have

$$\Gamma_{c}^{max} = \max_{\mathbf{x}} \max_{\mathbf{y}} \frac{\mathbf{y}^{H} \left(B^{-\frac{1}{2}}\right)^{H} \mathbf{H}_{c} \mathbf{K} \mathbf{x} \mathbf{x}^{H} \mathbf{K}^{H} \mathbf{H}_{c}^{H} B^{-\frac{1}{2}} \mathbf{y}}{\mathbf{y}^{H} \mathbf{y}}$$
(17)  
s.t:  $\mathbf{x}^{H} \mathbf{x} = 1$ 

 $\bullet\,$  Without changing the problem, we can constrain  $\|\mathbf{y}\|=1$  and get

$$\Gamma_{c}^{max} = \max_{\mathbf{x}} \max_{\mathbf{y}} \mathbf{y}^{H} \left( B^{-\frac{1}{2}} \right)^{H} \mathbf{H}_{c} \mathbf{K} \mathbf{x} \mathbf{x}^{H} \mathbf{K}^{H} \mathbf{H}_{c}^{H} B^{-\frac{1}{2}} \mathbf{y}$$
(18)  
**s.t:**  $\mathbf{x}^{H} \mathbf{x} = 1, \mathbf{y}^{H} \mathbf{y} = 1$   
 $\Leftrightarrow \Gamma_{c}^{max} = \max_{\mathbf{z}} \{ \max_{\mathbf{y}} \mathbf{y}^{H} \mathbf{z} \mathbf{z}^{H} \mathbf{y} \}$ (19)  
**s.t:**  $\mathbf{x}^{H} \mathbf{x} = 1, \mathbf{y}^{H} \mathbf{y} = 1$   
Where  $\mathbf{z} \triangleq \left( B^{-\frac{1}{2}} \right)^{H} \mathbf{H}_{c} \mathbf{K} \mathbf{x}$ 

#### **Orthogonal Beamforming III**

• The solution to the inner maximization of (19) is simply  $\mathbf{y} = \frac{\mathbf{z}}{\|\mathbf{z}\|}$ . Consequently, we have

$$\Gamma_{c}^{max} = \max_{\mathbf{z}} \frac{\mathbf{z}^{H} \mathbf{z} \mathbf{z}^{H} \mathbf{z}}{\mathbf{z}^{H} \mathbf{z}} = \max_{\mathbf{z}} \mathbf{z}^{H} \mathbf{z} = \max_{\mathbf{x}} \{\mathbf{x}^{H} \mathbf{A} \mathbf{x}\}$$
(20)  
s.t.  $\mathbf{x}^{H} \mathbf{x} = 1$   
Where  $\mathbf{A} = \mathbf{K}^{H} \mathbf{H}_{c}^{H} B^{-1} \mathbf{H}_{c} \mathbf{K}$ 

• Problem (20) can be recognized as finding the max eigen value  $\lambda_{max}(\mathbf{A})$ , which can be solved for  $\mathbf{x}$ . Once  $\mathbf{x}$  is found, we can obtain  $\mathbf{v}_c$  from (14) and then solve for  $\mathbf{u}_c$  as follows:

$$\mathbf{u}_{c} = B^{-\frac{1}{2}}\mathbf{y} = B^{-\frac{1}{2}}\frac{\mathbf{z}}{\|\mathbf{z}\|}$$
(21)  
Where  $\mathbf{z} = \left(B^{-\frac{1}{2}}\right)^{H}\mathbf{H}_{c}\mathbf{K}\mathbf{x}$  (22)

# Zero-forcing Beamforming I

- When N > M, it is possible to impose zero-forcing constraint on v<sub>c</sub> in order to eliminate interference to the D2D link.
- For the cellular link, we maximize its SINR and get the following optimization problem:

$$\max_{\mathbf{v}_c, \mathbf{u}_c} \Gamma_c = \max_{\mathbf{v}_c} \max_{\mathbf{u}_c} \frac{\mathbf{u}_c^H \Phi_c \mathbf{u}_c}{\mathbf{u}_c^H B \mathbf{u}_c}$$
(23)  
subject to:  $\mathbf{H}_{cd} \mathbf{v}_c = 0$  (24)  
 $\|\mathbf{v}_c\| = 1$  (25)

• Constraint (24) means  $\mathbf{v}_c$  lies in the null space of  $\mathbf{H}_{cd}$ . Let  $\mathbf{H}_{cd} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H$  be the singular decomposition of  $\mathbf{H}_{cd}$ . A basis of the null space of  $\mathbf{H}_{cd}$  is the last (N - R) columns of  $\mathbf{V}$ , denoted as  $\mathbf{v}_{R+1}, \mathbf{v}_{R+2}, \dots, \mathbf{v}_N$ , where R is the rank of  $\mathbf{H}_{cd}$ .

#### Zero-forcing Beamforming II • As a result, $\mathbf{v}_c$ is a linear combination of this basis:

$$\mathbf{v}_{c} = [\mathbf{v}_{R+1}, \mathbf{v}_{R+2}, \dots, \mathbf{v}_{N}] \mathbf{x} = \mathbf{K}' \mathbf{x}$$
(26)  
$$\Phi_{c} = \mathbf{H}_{c} \mathbf{v}_{c} \mathbf{v}_{c}^{H} \mathbf{H}_{c}^{H} = \mathbf{H}_{c} \mathbf{K}' \mathbf{x} \mathbf{x}^{H} \mathbf{K}'^{H} \mathbf{H}_{c}^{H}$$
(27)

Where  $\mathbf{x} = [x_1, x_2, ..., x_{N-R}]^T$ 

• Consequently, we have:

$$\max_{\mathbf{x},\mathbf{u}_{c}} \frac{\mathbf{u}_{c}^{H} \left(\mathbf{H}_{c} \mathbf{K}' \mathbf{x} \mathbf{x}^{H} \mathbf{K}'^{H} \mathbf{H}_{c}^{H}\right) \mathbf{u}_{c}}{\mathbf{u}_{c}^{H} B \mathbf{u}_{c}}$$
subject to:  $\|\mathbf{x}\| = 1$ 
(28)

- Problem (28) is identical to problem (16) in the Orthogonal Beamforming section with K' instead of K.
- Hence, the solution  $\mathbf{x}$  must be the max eigen vector corresponding to  $\lambda_{max}(\mathbf{A}')$ , where  $\mathbf{A}' = {\mathbf{K}'}^H \mathbf{H}_c^H B^{-1} \mathbf{H}_c \mathbf{K}'$ . The solutions for  $\mathbf{u}_c$  and  $\mathbf{v}_c$  can be obtained in the same way.

#### Tunable Beamforming I

 The zero-forcing solution can cause poor performance for the cellular link. Thus, it may be advantageous to add a *tunable* component outside of the null-space of H<sub>cd</sub>.

$$\mathbf{v}_{c} = \left[\mathbf{v}_{R}, \mathbf{v}_{R+1}, \mathbf{v}_{R+2}, \dots, \mathbf{v}_{N}\right] \left[\begin{array}{c} t\\ \mathbf{x} \end{array}\right] = \mathbf{K}'' \mathbf{x}'$$
(29)

$$\Phi_c = \mathbf{H}_c \mathbf{v}_c \mathbf{v}_c^H \mathbf{H}_c^H = \mathbf{H}_c \mathbf{K}'' \mathbf{x}' \mathbf{x}'^H \mathbf{K}''^H \mathbf{H}_c^H$$
(30)

Where  $\mathbf{v}_R$  corresponds to the smallest singular value of  $\mathbf{H}_{cd}$ , and  $t \in [0, 1]$  is the *tuning* parameter.

• For a fixed t, we can solve for x and  $u_c$  by maximizing the SINR:

$$\Gamma_{c}^{max} = \max_{\mathbf{x},\mathbf{u}_{c}} \Gamma_{c} = \max_{\mathbf{x}} \max_{\mathbf{u}_{c}} \frac{\mathbf{u}_{c}^{H} \left(\mathbf{H}_{c} \mathbf{K}'' \mathbf{x}' \mathbf{x}'^{H} \mathbf{K}''^{H} \mathbf{H}_{c}^{H}\right) \mathbf{u}_{c}}{\mathbf{u}_{c}^{H} B \mathbf{u}_{c}}$$
(31)  
s.t:  $\mathbf{x}'^{H} \mathbf{x}' = 1$ 

#### **Tunable Beamforming II**

• Following similar steps in the *Orthogonal Beamforming* section, we arrive at

$$\Gamma_c^{max} = \max_{\mathbf{x}} \{ \mathbf{x'}^H \mathbf{A''} \mathbf{x'} \}$$
(32)  
s.t.  $\mathbf{x'}^H \mathbf{x'} = 1$   
Where  $\mathbf{A''} = \mathbf{K''}^H \mathbf{H}_c^H B^{-1} \mathbf{H}_c \mathbf{K''}$ (33)

• Let 
$$\mathbf{A}'' = \begin{bmatrix} a_{11} & \mathbf{q}^H \\ \mathbf{q} & \mathbf{Q} \end{bmatrix}$$
, we get  

$$\Gamma_c^{max} = \max_{\mathbf{x}} \left\{ \mathbf{x}^H \mathbf{Q} \mathbf{x} + (\mathbf{q}^H \mathbf{x} + \mathbf{x}^H \mathbf{q})t + a_{11}t^2 \right\}$$
(P4)  
Subject to:  $\mathbf{x}^H \mathbf{x} = (1 - t^2)$ (34)

#### Tunable Beamforming III

 Problem (P4) can be solved using Lagrange method with the help of Wirtinger calculus. The solution is

$$\mathbf{x} = -t(\mathbf{Q} + \mu \mathbf{I})^{-1}\mathbf{q}$$
(35)

Here µ is the minimum real solution of the following equation

$$f(\mu) \triangleq \sum_{i=1}^{k} \frac{|p_i|^2}{(\lambda_i + \mu)^2} = \alpha$$
(36)

Where  $\mathbf{Q} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H$  is an eigen-value decomposition of  $\mathbf{Q}$ ,  $\mathbf{\Lambda} = \operatorname{diag}(\lambda_1, \lambda_2, \dots, \lambda_k), \ k = \operatorname{rank}(\mathbf{Q}), \ \alpha = \frac{1-t^2}{t^2}, \ \text{and}$  $\mathbf{p} \triangleq \mathbf{U}^H \mathbf{q} = [p_1, p_2, \dots, p_k]^T.$ 

### Tunable Beamforming IV



#### **Simulation Results**



PhuongBang Nguyen (UCSD)

## Outline

# 1 Introduction

- 2 Single Antenna Scenario (SISO)
- 3 Multiple Antenna Scenario (MIMO)

#### 4 References

#### **References I**

- H. Holma and A. Toskala, WCDMA for UMTS: HSPA Evolution and LTE. John Wiley & Sons, 2010.
- [2] A. Gjendemsjo, G. E. Oien, and D. Gesbert, "Binary power control for multi-cell capacity maximization," *IEEE 8th Workshop on Signal Processing Advances in Wireless Communications*, pp. 1–5, Jun 2007.
- [3] M. Chiang, "Geometric programming for communications systems," Foundations and Trends in Communications and Information Theory, vol. 2, pp. 1–156, Aug 2005.
- [4] B. Song, R. Cruz, and B. Rao, "Network duality for multiuser mimo beamforming networks and applications," *IEEE Transactions on Communications*, vol. 55, pp. 618–630, Mar 2007.

[5] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Transactions on Communications*, vol. 47, pp. 1458–1461, Oct 1999.