Cell throughput analysis of the Proportional Fair scheduler in the single cell environment

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IEEE Trans on Vehicular Tech, Mar 2007

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February 21, 2013

Outline

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Schedulers

- Round Robin: sequentially allocates resource to users. Loss in multiuser diversity.
- Min-max: maximize the minimum rate.
- **Proportional Fair** [1, 2]: allocates reasonable portion of the resource to all users while giving preference to the users with good channel condition.

Outline

System model

- A downlink multiuser system where the BS serves N users
- The received signal at user k is $P_k = |h_k|^2 P_t$.

$$h_k = \sqrt{c d_k^{-\alpha} s_k} m_k, \quad (1)$$

where c is constant, d_k is distant BS-user k, random variable s_k is for shadowing effect (log-normal with variance σ_s^2dB), m_k represents Rayleigh fading.

- The average received SNR of user k

$$\bar{Z}_k = \rho (D/d_k)^{\alpha} s_k, \quad (2)$$

where D is the radius of the cell, $\rho = cD^{-\alpha} P_t / P_n$ the average SNR at the cell edge.

Proportional Fair scheduler

- PF select user k^*

$$k^* = \arg\max_k \frac{R_k[n]}{\tilde{R}_k[n]}, \quad (3)$$

where $R_k[n]$ the instantaneous rate, $\tilde{R}_k[n]$ is the average throughput of user k

$$\tilde{R}_k[n+1] = \begin{cases} (1-\frac{1}{t_c})\tilde{R}_k[n] + \frac{1}{t_c}R_k[n] & k = k^* \\ (1-\frac{1}{t_c})\tilde{R}_k[n] & k \neq k^* \end{cases} \quad (4)$$

, where t_c is the time constant for the moving average.

Outline

Assumptions

- Users are distributed uniformly throughout the entire cell area.
- Every session is always active in the downlink direction.
- The distribution of channel gain of user k does not depend on time slot n and is constant for the slot duration.
- In this model, the ratio of the SNR to the average SNR is used.
- The feasible rate is a strictly monotonic increasing function of the SNR.
- Average throughput and average SNR are obtained by the time average.

Cell throughput of the PF scheduler

- Suppose the average rate of user k, $\tilde{R}_k[n]$, gets stable and stationary as time goes by

$$T_k = \lim_{n\to\infty} \tilde{R}_k[n] = \lim_{n\to\infty} E\{R_k[n] I_k\}, \tag{5}$$

with I_k is the indicator which equal 1 when the user is allocated.

- The preference metric is

$$\Gamma_k = \lim_{n\to\infty} \frac{Z_k[n]}{\tilde{Z}_k[n]} = \frac{Z_k}{\tilde{Z}_k}, \tag{6}$$

where Z_k, $\tilde{Z}_k$ are the instantaneous and the average SNR.

Cell throughput of the PF scheduler

- The longterm average throughput of user k is

$$T_k = Pr\{\Gamma_k > \Gamma_{k-}\}E\{R_k|\Gamma_k > \Gamma_{k-}\} = \int_{\xi(0)}^{\xi(\infty)} \xi(t) f_{\Gamma_k}(t) F_{\Gamma_{k-}}(t) dt, \quad (7)$$

where the instantaneous rate $R_k = \xi(\Gamma_k)$, $f_{\Gamma_k}(t)$ is the distribution of $\Gamma_k = \frac{Z_k}{\tilde{Z}_k}$, and $F_{\Gamma_{k-}}(t)$ is the distribution of the maximum Γ_j with $j = 1, \ldots, K$ and $j \neq k$.

Cell throughput of the PF scheduler

- Under Rayleigh fading, throughput of user k is

$$T_k = \int_{\xi(0)}^{\xi(\infty)} \xi(t) \frac{1}{\Gamma} \exp\left(-\frac{t}{\Gamma}\right) \left(1 - \exp\left(-\frac{t}{\Gamma}\right)\right)^{N-1} dt, \qquad (8)$$

where $\xi(t)$ is the rate function

PF - linear model

- The feasible rate is linearly proportional to the SNR $R_k = \beta W Z_k$. The average throughput

$$T_k = \frac{\beta W \bar{Z}_k}{N} \int_0^\infty t e^{-t} (1 - e^{-t})^{N-1} dt$$
$$= \left(\frac{\beta W}{N} M(N) \right) E_s(\bar{Z}_k)$$
$$= \left(\frac{\beta W}{N} M(N) \right) E_s(s_k) \rho \left(\frac{D}{d_k} \right)^\alpha , \quad (9)$$

with $\bar{Z}_k = \rho(D/d_k)^\alpha s_k$ and $M(N) = N \sum_{m=0}^{N-1} \binom{N-1}{m} \frac{(-1)^m}{(m+1)^2}$.

PF - linear model

- Taking average over the entire cell $E_A\{.\}$, the cell throughput is

$$\begin{aligned}\hat{T}_{cell} &= NE_A\{E_s\{T_k\}\} \\ &= \beta WN(M)E_s\{s_k\}\Omega_A^{-1}\int_A \rho\left(\frac{D}{d_k}\right)^{\alpha} dA \\ &= W\frac{2\rho\beta}{2-\alpha}\frac{1-\eta^{2-\alpha}}{1-\eta^2}\exp\left(\left(\frac{ln10}{10\sqrt{2}}\sigma_s\right)^2\right)M(N), \qquad (10)\end{aligned}$$

by using $E_s\{s_k\} = \exp(((ln10)/10\sqrt{2}\sigma_s)^2)$.

PF - logarithmic model

- The rate to user k is $R_k = W \log_2\left(1 + \frac{Z_k}{K}\right)$, where K is a constant depending on the system design and the target BER. Similarly,

$$T_k = \frac{W}{ln2} \int_0^{\infty} ln\left(1 + \frac{\bar{Z}_k}{K} t\right) e^{-t}(1 - e^{-t})^{N-1} dt$$

$$= \frac{W}{ln2} \sum_{m=0}^{N-1} \binom{N-1}{m} \frac{(-1)^m}{(m+1)} \exp\left(\frac{K}{\bar{Z}_k}(m+1)\right) Ei\left(\frac{K}{\bar{Z}_k}(m+1)\right)$$

$$\simeq W\nu_1 \sum_{m=0}^{N-1} \binom{N-1}{m} \frac{(-1)^m}{(m+1)} ln\left(1 + \frac{\nu_2}{K(m+1)} \bar{Z}_k\right). \quad (11)$$

where $\int_0^{\infty} ln(1 + at)e^{-bt} dt = \frac{1}{a} \exp(b/a) Ei(b/a)$, the parameters $\nu_1 = 1.4$ and $\nu_2 = 0.82$.

PF - logarithmic model

- Taking expectation over shadowing fading and average over the entire cell area

$$T_{cell} \simeq N\nu_1 \sum_{m=0}^{N-1} \binom{N-1}{m} \frac{(-1)^m}{(m+1)} (B_m + \nu_3), \quad (12)$$

where B_m is defined as

$$B_m = \frac{2}{D^2} \int_0^D r \ln \left(1 + b_m \left(\frac{D}{r} \right)^{\alpha} \right) dr, \quad (13)$$

with $b_m = (\nu_2 \rho)/(K(m+1))$. Note B_m can be exactly calculate for α integer. When $\alpha = 4$,

$$B_m = \ln(1 + b_m) + 2b_m^{0.5} \arctan b_m^{-0.5}$$

Outline

MIMO systems

- n_T transmit antennas, n_R receive antennas, $n_T = n_R = n_A$. The signal received by RA_j is

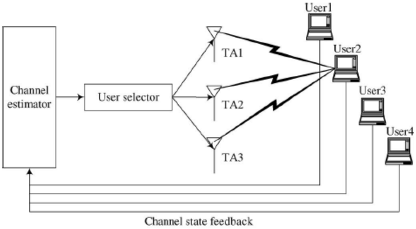
$$y_j = \sum_{i=1}^{n_T} h_{ij} x_i + n_j, \tag{14}$$

where n_j denotes noise. Then, $Z_k^{(j)}$ has exponential distribution.

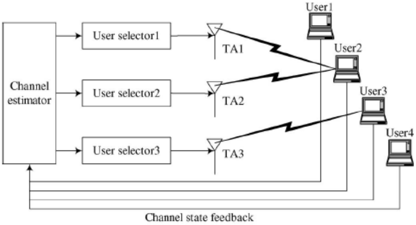
- The cell throughput in logarithmic rate model is given

$$T_{cell} = N n_A \nu_1 \sum_{m=0}^{N-1} \binom{N-1}{m} \frac{(-1)^m}{(m+1)} (B_m + \nu_3). \tag{15}$$

PF in MIMO systems



(a)



(b)

Simulations results

- Single cell $D = 1$km.
- Transmit power $P_t = 10$W.
- pathloss exponent $\alpha = 4$, shadow fading $\sigma_s = 8$dB.
- The median SNR at the cell edge $\rho = 0$dB. System efficiency factor $K = 8$dB.
- Two user 100, 200m from the BS.

Time average vs. moving average

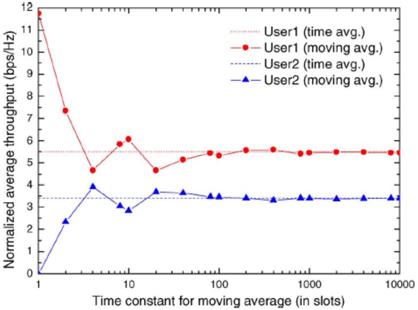
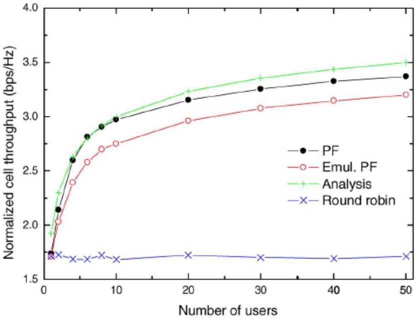


Fig. 5. Comparison of the moving average with the time average.

Time average vs. moving average



Time average vs. moving average

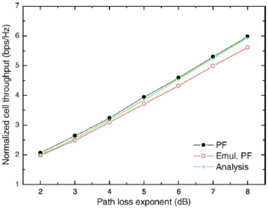


Fig. 8. Effect of the path-loss exponent on the cell throughput: $\sigma_s = 8$ dB, $\rho = 0$ dB, $K = 8$ dB, and $N = 30$.

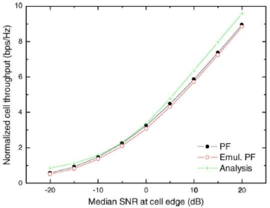


Fig. 10. Effect of the median SNR at the cell edge on the cell throughput: $\alpha = 4$, $\sigma_s = 8$ dB, $K = 8$ dB, and $N = 30$.

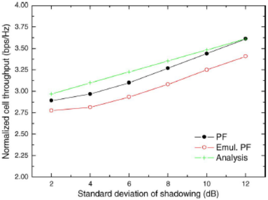


Fig. 9. Effect of the standard deviation of shadowing on the cell throughput: $\alpha = 4$, $\rho = 0$ dB, $K = 8$ dB, and $N = 30$.

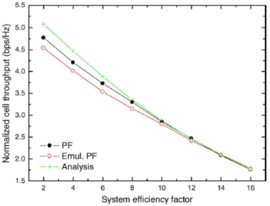


Fig. 11. Effect of the system-efficiency factor on the cell throughput: $\alpha = 4.0$, $\sigma_s = 8$ dB, $\rho = 0$ dB, and $N = 30$.

Time average vs. moving average

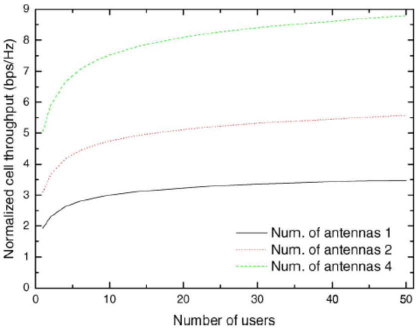


Fig. 14. Normalized cell throughput with multiple antennas: $\alpha = 4$, $\sigma_s = 8$ dB, $K = 8$ dB, and $\rho = 0$ dB.

Outline

Conclusion

- Question: Is PF the best?
- We look for an alternative/complementary algorithm.
  - Guarantee fairness.
  - Have good performance.
  - Be practical.

References

- F. P. Kelly, A. K. Maulloo, and D. K. Tan, “Rate control for communication networks: shadow prices, proportional fairness and stability,” *Journal of the Operational Research society*, vol. 49, no. 3, pp. 237–252, 1998.
- J.-G. Choi and S. Bahk, “Cell-throughput analysis of the proportional fair scheduler in the single-cell environment,” *Vehicular Technology, IEEE Transactions on*, vol. 56, pp. 766 –778, march 2007.

Thank you!

Questions?