Homework # 4

The first two problems are due next Wednesday (2/25) and the computer assignment is due in two weeks on 3/04.

1. Prove the following result (Taken from the document "An Introduction to Compressive Sensing" at http://cnx.org/content/col11133/1.5/)

   Suppose that the $n \times m$ measurement matrix A satisfies the RIP of order 2K, and let $h \in R^m$, $h \neq 0$ be arbitrary. Let Λ_0 be any subset of $\{1, 2, ..., m\}$ such that $|\Lambda_0| \leq K$. Define Λ_1 as the index set corresponding to the K entries of $h_{\Lambda_0^c}$ with largest magnitude and set $\Lambda = \Lambda_0 \cup \Lambda_1$. Then

   $$\|h_\Lambda\|_2 \leq \alpha \frac{\|h_{\Lambda_0^c}\|_1}{\sqrt{K}} + \beta \frac{| < Ah_\Lambda, Ah > |}{\|h_\Lambda\|_2},$$

   where

   $$\alpha = \frac{\sqrt{2}\delta_{2K}}{1 - \delta_{2K}}, \quad \beta = \frac{1}{1 - \delta_{2K}}$$

2. Show the following (Taken from the same document)

   Suppose that A satisfies the RIP of order K with constant δ_K. Let γ be a positive integer. Then A satisfies the RIP of order $K' = \gamma \lfloor \frac{K}{2} \rfloor$ with constant $\delta_{K'} < \gamma \delta_K$, where $\lfloor . \rfloor$ denotes the floor operator.

3. Matlab Computer Study

   (a) Conduct computer experiments to compare the performance of re-weighted ℓ_1 norm (Candes, Wakin and Boyd) and re-weighted ℓ_2 norm (Chartrand) based sparse signal recovery methods with algorithms from the previous home works.

   (b) Please include a complexity (flop count) study.

4. An interesting reference on Restricted Isometry Constants for your reference. "New Bounds for Restricted isometry Constants," by T. T. Cai, L. Wang, and G. Xu, IEEE Transactions on Information Theory, Sept. 2010.