Homework # 2 Solution

Problem 1: Show that for a full rank A matrix, $\delta(A)$ is strictly greater than zero.

Since A is full rank, the smallest eigenvalue λ_{min} of AA^T is strictly greater than 0. This implies $0 < \lambda_{min} = \inf_v \frac{v^T AA^T v}{\|v\|^2}$. Furthermore

$$\frac{v^T AA^T v}{\|v\|^2} = \frac{\|A^T v\|^2}{\|v\|^2} = \frac{\sum_{l=1}^{m} |a_l^T v|^2}{\|v\|^2} \leq m \max_l \frac{|a_l^T v|^2}{\|v\|^2} = m\delta(A, v)$$

Taking the inf over v on both sides, we get

$$0 < \lambda_{min} \leq m\delta(A) \text{ or } \delta(A) \geq \frac{\lambda_{min}}{m} > 0.$$

Problem 2: Equivalent of theorem 4.3 for Matching Pursuit. Consider the system of equations $Ax = b$, where A is $n \times m$ with $n < m$ and A full rank. If a solution x^* exist obeying $\|x^*\|_0 < \frac{1}{2}(1 + \frac{1}{\mu(A})$, then MP run with a threshold of zero is guaranteed to select only the columns with the nonzero weights. (Make sure to understand the difference compared to the result for OMP).

The proof does not change. We can conclude that the MP algorithm, like the OMP algorithm, will select at each step one of the columns corresponding to the non-zero entries. While in OMP, once a column l has been selected, it is not reselected since $a_l^T b_p$ is zero in the subsequent steps. As a result, a new correct column is selected at each iteration and the algorithm will converge in r steps, where r is the sparsity. In the MP however, once a column l has been selected, it can be reselected since $a_l^T b_p$ is not zero in the subsequent, but not immediate, step. Also, because of the way b_p is computed, the error may take a while to converge to zero. So, even though we will be selecting columns from the correct subset at each step, the columns may be selected multiple times and the number of iterations can be large.