

Homework # 1

The first two problems are due next Wednesday (1/21) and the computer assignment is due in two weeks on 1/28.

1. Let X_1 and X_2 be full column rank $N \times M$ matrices that have the same range space. Show that the projection matrices $X_1X_1^+$ and $X_2X_2^+$ are the same.
2. P_M and P_S are orthogonal projection matrices onto subspaces of dimensions M and S respectively in C^N . Is P_MP_S an orthogonal projection matrix? If yes, prove your answer. If not, explain why not and identify conditions under which it is an orthogonal projection matrix.
3. Matlab Computer Study
 - (a) Conduct computer experiments to generate and verify the results for Matching Pursuit type algorithms in the textbook by Elad (Figures 3.5 and 3.6). For the non-zero entries, in addition to the process described in the book, consider the following three options: $N(0, 1)$, ± 1 , and Cauchy distribution. Generate similar plots for these three options.
 - (b) Please include a complexity (flop count) study. So please be efficient in your algorithm code.
 - (c) Plot a histogram of the mutual coherence ($\mu(A)$) of the A matrices as well as the upper bound of the $\frac{1}{2}(1 + \frac{1}{\mu(A)})$ to get an idea of the nature of the uniqueness bounds.
 - (d) Derive an analytical expression for the density function of $\mu(A)$ for the Gaussian A matrix and compare with the experimental result (Optional)