9.28 A microwave radio channel has a frequency response

\[ C(f) = 1 + 0.3 \cos 2\pi f T \]

Determine the frequency-response characteristic of the transmitting and receiving filters that yield zero ISI at a rate of \( 1/T \) symbols/s and have a 50 percent excess bandwidth. Assume that the additive noise spectrum is flat.

9.29 \( M = 4 \) PAM modulation is used for transmitting at a bit rate of 9600 bits/s on a channel having a frequency response

\[ C(f) = \frac{1}{1 + j(f/2400)} \]

for \(|f| \leq 2400\), and \( C(f) = 0 \) otherwise. The additive noise is zero-mean white Gaussian with power spectral density \( \frac{1}{2} N_0 \) W/Hz. Determine the (magnitude) frequency-response characteristic of the optimum transmitting and receiving filters.

9.30 Use the Cauchy–Schwarz inequality to show that the transmitter and receiver filters given by Equation 9.2–83 minimize the noise-to-signal ratio \( \sigma_n^2 \nu / d^2 \), where \( \sigma_n^2 \) is the noise power given by Equation 9.2–77, where \( S_{nn}(f) = N_0/2 \).

9.31 Suppose that a channel frequency response is given as

\[ C(f) = \begin{cases} 
1 & |f| \leq W/2 \\
\frac{W}{2} & W/2 < |f| < W 
\end{cases} \]

Determine the loss in SNR incurred, as given by Equations 9.2–87 and 9.2–88, for the filters given by the corresponding Equations 9.2–79 and 9.2–83, respectively. Which filters result in a smaller loss?

9.32 In a binary PAM system, the input to the detector is

\[ y_m = a_m + n_m + i_m \]

where \( a_m = \pm 1 \) is the desired signal, \( n_m \) is a zero-mean Gaussian random variable with variance \( \sigma_n^2 \), and \( i_m \) represents the ISI due to channel distortion. The ISI term is a random variable that takes the values \(-\frac{1}{2}, 0, \) and \( \frac{1}{2} \) with probabilities \( \frac{1}{4}, \frac{1}{2}, \) and \( \frac{1}{4} \), respectively. Determine the average probability of error as a function of \( \sigma_n^2 \).

9.33 In a binary PAM system, the clock that specifies the sampling of the correlator output is offset from the optimum sampling time by 10 percent.

a. If the signal pulse used is rectangular, determine the loss in SNR due to the mistiming.

b. Determine the amount of ISI introduced by the mistiming and determine its effect on performance.
Chapter Nine: Digital Communication Through Band-Limited Channels

(noise-free) output of the demodulator is $\sqrt{E_b}$ at $t = T$, $\sqrt{E_b}/4$ at $t = 2T$, and zero for $t = kT$, $k > 2$, where $E_b$ is the signal energy and $T$ is the signaling interval.

a. Determine the average probability of error, assuming that the two signals are equally probable and the additive noise is white and Gaussian.

b. By plotting the error probability obtained in (a) and that for the case of no ISI, determine the relative difference in SNR of the error probability of $10^{-6}$.

9.40 Derive the expression in Equation 9.5–5 for the coefficients in the feedback filter of the DFE.

9.41 Binary PAM is used to transmit information over an unequalized linear filter channel. When $a = 1$ is transmitted, the noise-free output of the demodulator is

$$x_m = \begin{cases} 
0.3 & m = 1 \\
0.9 & m = 0 \\
0.3 & m = -1 \\
0 & \text{otherwise}
\end{cases}$$

a. Design a three-tap zero-forcing linear equalizer so that the output is

$$q_m = \begin{cases} 
1 & m = 0 \\
0 & m = \pm 1
\end{cases}$$

b. Determine $q_m$ for $m = \pm 2, \pm 3$, by convolving the impulse response of the equalizer with the channel response.

9.42 The transmission of a signal pulse with a raised cosine spectrum through a channel results in the following (noise-free) sampled output from the demodulator:

$$x_k = \begin{cases} 
-0.5 & k = -2 \\
0.1 & k = -1 \\
1 & k = 0 \\
-0.2 & k = 1 \\
0.05 & k = 2 \\
0 & \text{otherwise}
\end{cases}$$

a. Determine the tap coefficients of a three-tap linear equalizer based on the zero-forcing criterion.

b. For the coefficients determined in (a), determine the output of the equalizer for the case of the isolated pulse. Thus, determine the residual ISI and its span in time.

9.43 A nonideal band-limited channel introduces ISI over three successive symbols. The (noise-free) response of the matched filter demodulator sampled at the sampling time $kT$ is

$$\int_{-\infty}^{\infty} s(t)s(t - kT) \, dt = \begin{cases} 
E_b & k = 0 \\
0.9E_b & k = \pm 1 \\
0.1E_b & k = \pm 2 \\
0 & \text{otherwise}
\end{cases}$$
a. Determine the tap coefficients of a three-tap linear equalizer that equalizes the channel (received signal) response to an equivalent partial-response (duobinary) signal

\[ y_k = \begin{cases} E_b & k = 0, 1 \\ 0 & \text{otherwise} \end{cases} \]

b. Suppose that the linear equalizer in (a) is followed by a Viterbi sequence detector for the partial signal. Give an estimate of the error probability if the additive noise is white and Gaussian, with power spectral density \( \frac{1}{2} N_0 \) W/Hz.

9.44 Determine the tap weight coefficients of a three-tap zero-forcing equalizer if the ISI spans three symbols and is characterized by the values \( x(0) = 1, x(-1) = 0.3, x(1) = 0.2 \). Also determine the residual ISI at the output of the equalizer for the optimum tap coefficients.

9.45 In line-of-sight microwave radio transmission, the signal arrives at the receiver via two propagation paths: the direct path and a delayed path that occurs due to signal reflection from surrounding terrain. Suppose that the received signal has the form

\[ r(t) = s(t) + \alpha s(t - T) + n(t) \]

where \( s(t) \) is the transmitted signal, \( \alpha \) is the attenuation (\( \alpha < 1 \)) of the secondary path, and \( n(t) \) is AWGN.

a. Determine the output of the demodulator at \( t = T \) and \( t = 2T \) that employs a filter matched to \( s(t) \).

b. Determine the probability of error for a symbol-by-symbol detector if the transmitted signal is binary antipodal and the detector ignores the ISI.

c. What is the error rate performance of a simple (one-tap) DFE that estimates \( \alpha \) and removes the ISI? Sketch the detector structure that employs a DFE.

9.46 Repeat Problem 9.41 using the MSE as the criterion for optimizing the tap coefficients. Assume that the noise power spectral density is 0.1 W/Hz.

9.47 In a magnetic recording channel, where the readback pulse resulting from a positive transition in the write current has the form

\[ p(t) = \left[ 1 + \left( \frac{2t}{T_{50}} \right)^2 \right]^{-1} \]

a linear equalizer is used to equalize the pulse to a partial response. The parameter \( T_{50} \) is defined as the width of the pulse at the 50 percent amplitude level. The bit rate is \( 1/T_b \) and the ratio of \( T_{50}/T_b = \Delta \) is the normalized density of the recording. Suppose the pulse is equalized to the partial-response values

\[ x(nT) = \begin{cases} 1 & n = -1, 1 \\ 2 & n = 0 \\ 0 & \text{otherwise} \end{cases} \]

where \( x(t) \) represents the equalized pulse shape.

a. Determine the spectrum \( X(f) \) of the band-limited equalized pulse.

b. Determine the possible output levels at the detector, assuming that successive transitions can occur at the rate \( 1/T_b \).
e. Evaluate and compare the exact values of the output SNR for the three-tap and infinite-tap DFE in the special cases where $N_0 = 0.1$ and 0.01. Comment on how well the three-tap equalizer performs relative to the infinite-tap equalizer.

**FIGURE P9.52**

9.53 A pulse and its (raised cosine) spectral characteristic are shown in Figure P9.53. This pulse is used for transmitting digital information over a band-limited channel at a rate $1/T$ symbols/s.

a. What is the roll-off factor $\beta$?

b. What is the pulse rate?

c. The channel distorts the signal pulses. Suppose the sampled values of the filtered received pulse $x(t)$ are as shown in Figure P9.53c. It is obvious that there are five interfering signal components. Give the sequence of $+1$s and $-1$s that will cause the largest (destructive or constructive) interference and the corresponding value of the interference (the peak distortion).

d. What is the probability of occurrence of the worst sequence obtained in (c), assuming that all binary digits are equally probable and independent?

**FIGURE P9.53**

9.54 A time-dispersive channel having an impulse response $h(t)$ is used to transmit four-phase PSK at a rate $R = 1/T$ symbols/s. The equivalent discrete-time channel is shown in