Tufts (1965), Smith (1965), and Berger and Tufts (1967). "Faster than Nyquist" transmission has been studied by Mazo (1975) and Foschini (1984).

Channel equalization for digital communications was developed by Lucky (1965, 1966), who focused on linear equalizers that were optimized using the peak distortion criterion. The mean-square-error criterion for optimization of the equalizer coefficients was proposed by Widrow (1966).

Decision-feedback equalization was proposed and analyzed by Austin (1967). Analyses of the performance of the DFE can be found in the papers by Monsen (1971), George et al. (1971), Price (1972), Salz (1973), Duttweiler et al. (1974), and Altekar and Beaulieu (1993).

The use of the Viterbi algorithm as the optimal maximum-likelihood sequence estimator for symbols corrupted by ISI was proposed and analyzed by Forney (1972) and Omura (1971). Its use for carrier-modulated signals was considered by Ungerboeck (1974) and MacKenchnie (1973).

The use of iterative MAP algorithms in suppressing ISI in coded systems, called turbo equalization, represents a major new advance in suppression of intersymbol interference in signal transmission through band-limited channels. It is anticipated that iterative MAP equalization algorithms will be incorporated in future communication systems. The implementation of turbo equalization, described in the paper by Hagenauer et al. (1999), is the first attempt at implementing an iterative MAP equalization algorithm in a coded system.

PROBLEMS

9.1 A channel is said to be *distortionless* if the response $y(t)$ to an input $x(t)$ is $Kx(t-t_0)$, where K and t_0 are contants. Show that if the frequency response of the channel is $A(f)e^{j\theta(f)}$, where $A(f)$ and $\theta(f)$ are real, the necessary and sufficient conditions for distortionless transmission are $A(f) = K$ and $\theta(f) = 2\pi f t_0 \pm n\pi$, $n = 0, 1, 2, \ldots$.

9.2 The raised cosine spectral characteristic is given by Equation 9.2–26.

a. Show that the corresponding impulse response is

$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\beta\pi t/T)}{1 - 4\beta^2 t^2/T^2}$$

b. Determine the Hilbert transform of $x(t)$ when $\beta = 1$.

c. Does $\hat{x}(t)$ possess the desirable properties of $x(t)$ that make it appropriate for data transmission? Explain.

d. Determine the envelope of the SSB suppressed-carrier signal generated from $x(t)$.

9.3 *a*. Show that (Poisson sum formula)

$$x(t) = \sum_{k=-\infty}^{\infty} g(t)h(t-kT) \Rightarrow X(f) = \frac{1}{T}\sum_{n=-\infty}^{\infty} H\left(\frac{n}{T}\right) G\left(f - \frac{n}{T}\right)$$

5. Show that in part 4, if noise powers are equal, then $\alpha = 1$, and determine the error probability in this case. How does this system compare with a system that has only one antenna, i.e., receives only $r_1(t)$?

4.54 A communication system employs binary antipodal signals with

$$s_1(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

and $s_2(t) = -s_1(t)$. The received signal consists of a direct component, a scattered component, and the additive white Gaussian noise. The scattered component is a delayed version of the basic signal times a random amplification A. In other words, we have $r(t) = s(t) + As(t-1) + n(t)$, where $s(t)$ is the transmitted message, A is an exponential random variable, and $n(t)$ is a white Gaussian noise with a power spectral density of $N_0/2$. It is assumed that the time delay of the multipath component is constant (equal to 1) and A and $n(t)$ are independent. The two messages are equiprobable and

$$f_A(a) = \begin{cases} e^{-a} & a > 0 \\ 0 & \text{otherwise} \end{cases}$$

1. What is the optimal decision rule for this problem? Simplify the resulting rule as much as you can.
2. How does the error probability of this system compare with the error probability of a system which does not involve multipath? Which one has a better performance?

4.55 A binary communication system uses equiprobable signals $s_1(t)$ and $s_2(t)$

$$s_1(t) = \sqrt{2\mathcal{E}_b}\,\phi_1(t)\cos(2\pi f_c t)$$
$$s_2(t) = \sqrt{2\mathcal{E}_b}\,\phi_2(t)\cos(2\pi f_c t)$$

for transmission of two equiprobable messages. It is assumed that $\phi_1(t)$ and $\phi_2(t)$ are orthonormal. The channel is AWGN with noise power spectral density of $N_0/2$.

1. Determine the optimal error probability for this system, using a coherent detector.
2. Assuming that the demodulator has a phase ambiguity between 0 and θ $(0 \le \theta \le \pi)$ in carrier recovery, and employs the same detector as in part 1, what is the resulting worst-case error probability?
3. What is the answer to part 2 in the special case where $\theta = \pi/2$?

4.56 In this problem we show that the volume of an n-dimensional sphere with radius R, defined by the set of all $\boldsymbol{x} \in \mathbb{R}^n$ such that $\|\boldsymbol{x}\| \le R$, is given by $V_n(R) = B_n R^n$, where

$$B_n = \frac{\pi^{\frac{n}{2}}}{\Gamma\left(\frac{n}{2}+1\right)}$$

1. Using change of variables, show that

$$V_n(R) = \underset{x_1^2+x_2^2+\cdots+x_n^2 \le R^2}{\int\int\cdots\int} dx_1\,dx_2\ldots dx_n = B_n R^n$$

where B_n is the volume on an n-dimensional sphere of radius 1, i.e., $B_n = V(1)$.

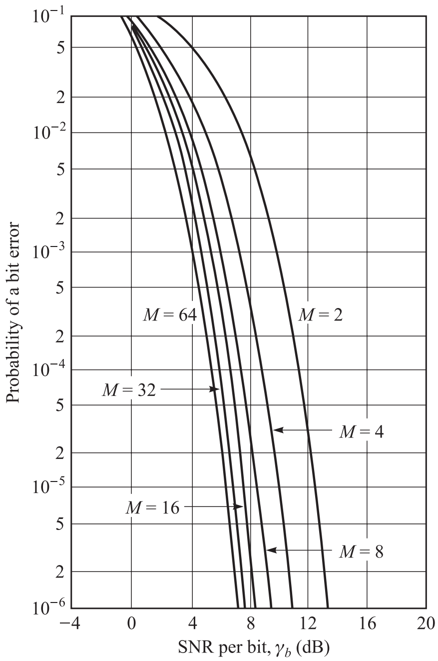

FIGURE 4.4–1
Probability of bit error for orthogonal signaling.

given by

$$\Delta f = \frac{l}{2T} \tag{4.4–13}$$

for a positive integer l. For this value of frequency separation the error probability of M-ary FSK is given by Equation 4.4–10.

Note that in the binary FSK signaling, a frequency separation that guarantees orthogonality does not minimize the error probability. In Problem 4.18 it is shown that the error probability of binary FSK is minimized when the frequency separation is of the form

$$\Delta f = \frac{0.715}{T} \tag{4.4–14}$$

A Union Bound on the Probability of Error in Orthogonal Signaling

The union bound derived in Section 4.2–3 states that

$$P_e \leq \frac{M-1}{2} e^{-\frac{d^2_{\min}}{4N_0}} \tag{4.4–15}$$

In orthogonal signaling $d_{\min} = \sqrt{2\mathcal{E}}$, therefore,

$$P_e \leq \frac{M-1}{2} e^{-\frac{\mathcal{E}}{2N_0}} < M e^{-\frac{\mathcal{E}}{2N_0}} \tag{4.4–16}$$

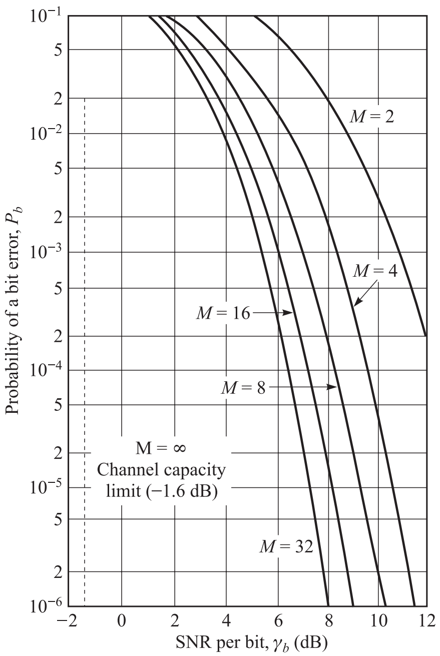

FIGURE 4.5–2
Probability of a bit error for noncoherent detection of orthogonal signals.

probability of a bit error P_b can be made arbitrarily small provided that the SNR per bit is greater than the Shannon limit of -1.6 dB. The cost for increasing M is the bandwidth required to transmit the signals. For M-ary FSK, the frequency separation between adjacent frequencies is $\Delta f = 1/T_s$ for signal orthogonality. The bandwidth required for the M signals is $W = M\,\Delta f = M/T_s$.

4.5–4 Probability of Error for Envelope Detection of Correlated Binary Signals

In this section, we consider the performance of the envelope detector for binary, equiprobable, and equal-energy correlated signals. When the two signals are correlated, we have

$$\boldsymbol{s}_{ml} \cdot \boldsymbol{s}_{m'l} = \begin{cases} 2\mathcal{E}_s & m = m' \\ 2\mathcal{E}_s \rho & m \neq m' \end{cases} \qquad m, m' = 1, 2 \tag{4.5–48}$$

where ρ is the complex correlation between the lowpass equivalent signals. The detector bases its decision on the envelopes $|\boldsymbol{r}_l \cdot \boldsymbol{s}_{1l}|$ and $|\boldsymbol{r}_l \cdot \boldsymbol{s}_{2l}|$, which are correlated (statistically dependent). Assuming that $s_1(t)$ is transmitted, these envelopes are given by

$$\begin{aligned} R_1 &= |\boldsymbol{r}_l \cdot \boldsymbol{s}_{1l}| = |2\mathcal{E}_s e^{j\phi} + \boldsymbol{n}_l \cdot \boldsymbol{s}_{1l}| \\ R_2 &= |\boldsymbol{r}_l \cdot \boldsymbol{s}_{2l}| = |2\mathcal{E}_s \rho e^{j\phi} + \boldsymbol{n}_l \cdot \boldsymbol{s}_{2l}| \end{aligned} \tag{4.5–49}$$