Array Design

The chosen design parameters are $N = 48$ and $\frac{d}{\lambda} = \frac{2}{3}$

Explanations: We satisfy the given constraints as follows:
1. *The array is to used for steering towards broadside over the range (*$60°$*,* $120°$*).*
Thus,

$$\frac{d}{\lambda} \leq \frac{1}{1 + \cos 60°} = \frac{2}{3}$$

3. *The BF has a beamwidth of* $20°$ *when steered to* $60°$*. Use the passband edges to determine the beamwidth.*
Let the passband edge be at ψ_p. Then, we have

$$\psi_t - \psi_p = 2\pi \frac{d}{\lambda} \cos B \tag{1}$$

$$\psi_t + \psi_p = 2\pi \frac{d}{\lambda} \cos b \tag{2}$$

where, $\psi_t = 2\pi \frac{d}{\lambda} \cos 60°$ and B and b are beam edges in theta-space when the beam is steered to $60°$. Since beamwidth is $20°$, we have

$$B - b = \frac{\pi}{9} = \cos^{-1}\left\{\frac{\lambda}{2\pi d}(\psi_t - \psi_p)\right\} - \cos^{-1}\left\{\frac{\lambda}{2\pi d}(\psi_t + \psi_p)\right\} \tag{3}$$

We solve the above equation for the passband edge, ψ_p

2. *The BF former has a low pass filter shape when steered towards broadside with a passband ripple of .01 and stopband ripple of .001. The transition bandwidth is* $5°$*. Use Parks-McClellan approach for filter design*
We shall use the above information to find the stopband edge ψ_s. When steered to broadside i.e $\psi_t = 0°$, let x be the corresponding passband edge in theta-space, measured from broadside. Thus,

$$\psi_p = 2\pi \frac{d}{\lambda} \cos(90° - x) \tag{4}$$

$$\psi_p \frac{\lambda}{2\pi d} = \sin x \tag{5}$$

$$x = sin^{-1}\left\{\psi_p \frac{\lambda}{2\pi d}\right\} \tag{6}$$

The stopband edge is then given by,

$$\psi_s = 2\pi \frac{d}{\lambda} \cos(90° - x - 5°) \tag{7}$$

Using, ψ_p and ψ_s we then design the Parks-McClellan filter. The number of array elements is chosen so as to constrain the ripples in passband to 0.01 and

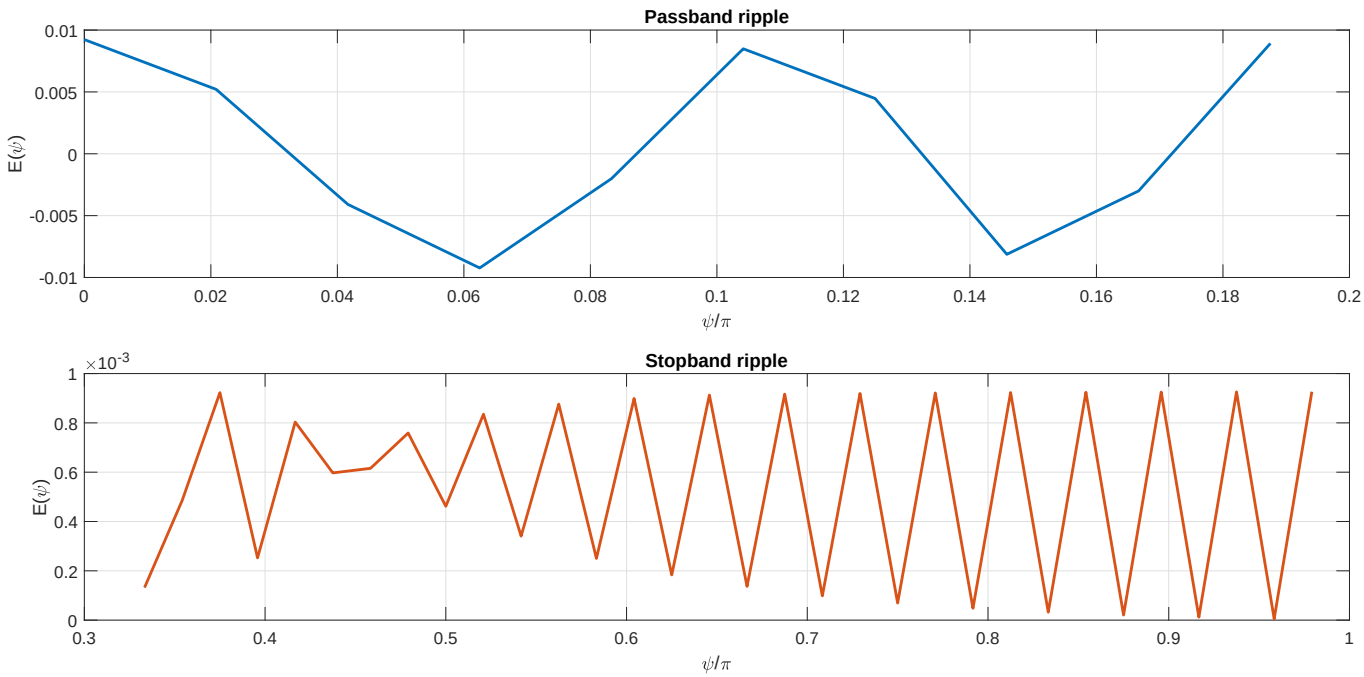
stopband to 0.001. This phenomena starts happening for $N \geq 48$. Thus, we chose $N = 48$.

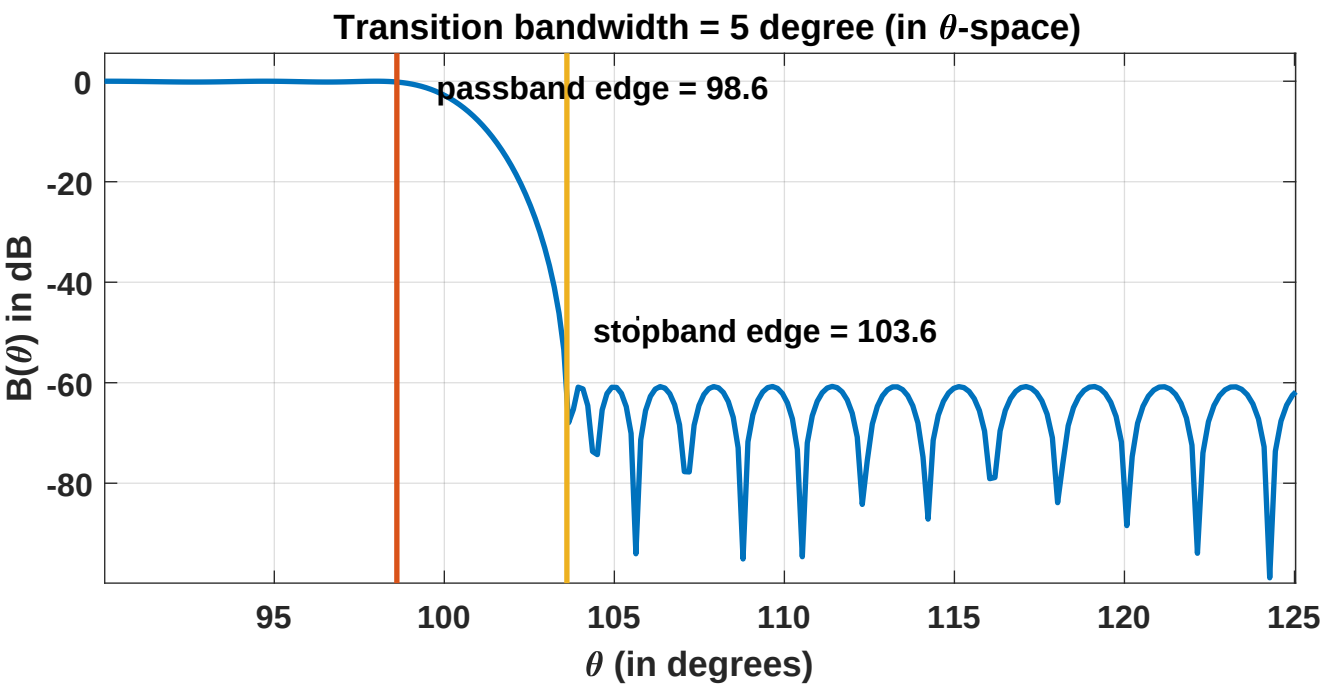
Code:

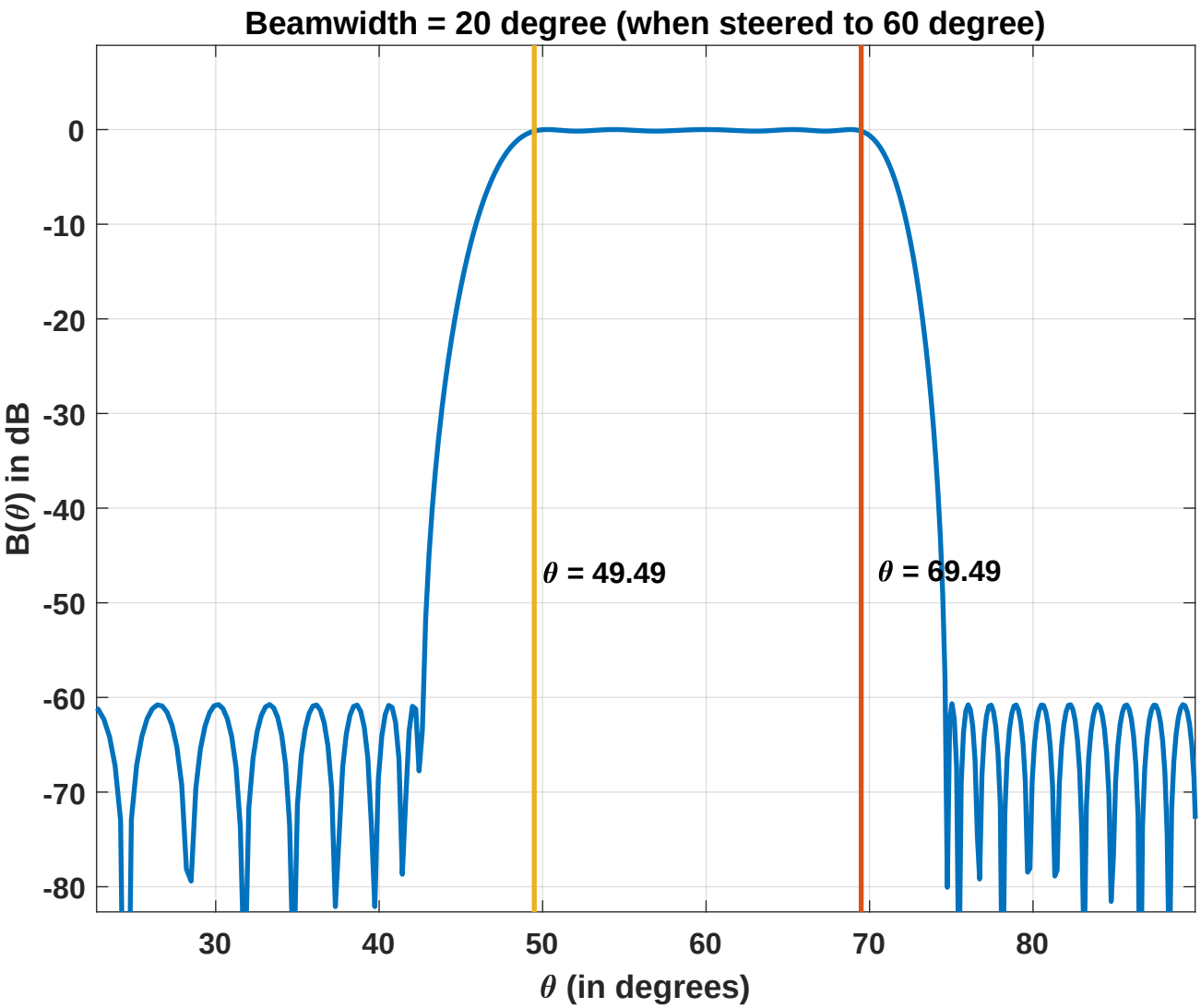
```
% Homework 2: Design Problem
close all
%% params

% Fixed
d_lambda = 2/3;
psi = -2*pi*d_lambda:0.01:2*pi*d_lambda;
theta = flip(acos(3*psi/(4*pi)));
%wp = 0.1;
%ws = 0.2;

% Solving subpart 3 to find wp
syms x
% to ensure beamwidth=20 degree when steer=60 degree
solu = vpasolve(acos(0.5-x*3/(4*pi))-acos(0.5+x*3/(4*pi)) == pi/9, x);
wp = double(solu)/pi;

% Using wp and subpart 2 to find ws
wp_thet = asin(pi*wp*3/(4*pi));
ws = 4*sin(wp_thet+5*pi/180)/3;

% Design params
N = 48;
ford = N-1; %filter order


%% Compute final beampattern
b = firpm(ford,[0 wp ws 1], [1 1 0 0], [1 10]);
freqz(b);
%wts = ifft(h);
wts = b;
[psi_mat, arg_mat] = meshgrid(psi, -(N-1)/2:(N-1)/2);
v_psi = exp(1i*psi_mat.*arg_mat);
B_psi = conj(wts)*v_psi/sum(wts);
B_psi_db = 10*log10(abs(B_psi).^2);
B_theta_db = flip(B_psi_db);

%% Filter response in Passband and Stopband
pb_ripple = max(abs(abs(h(w<=wp*pi))-1))
sb_ripple = max(abs(h(w>=ws*pi)))
```

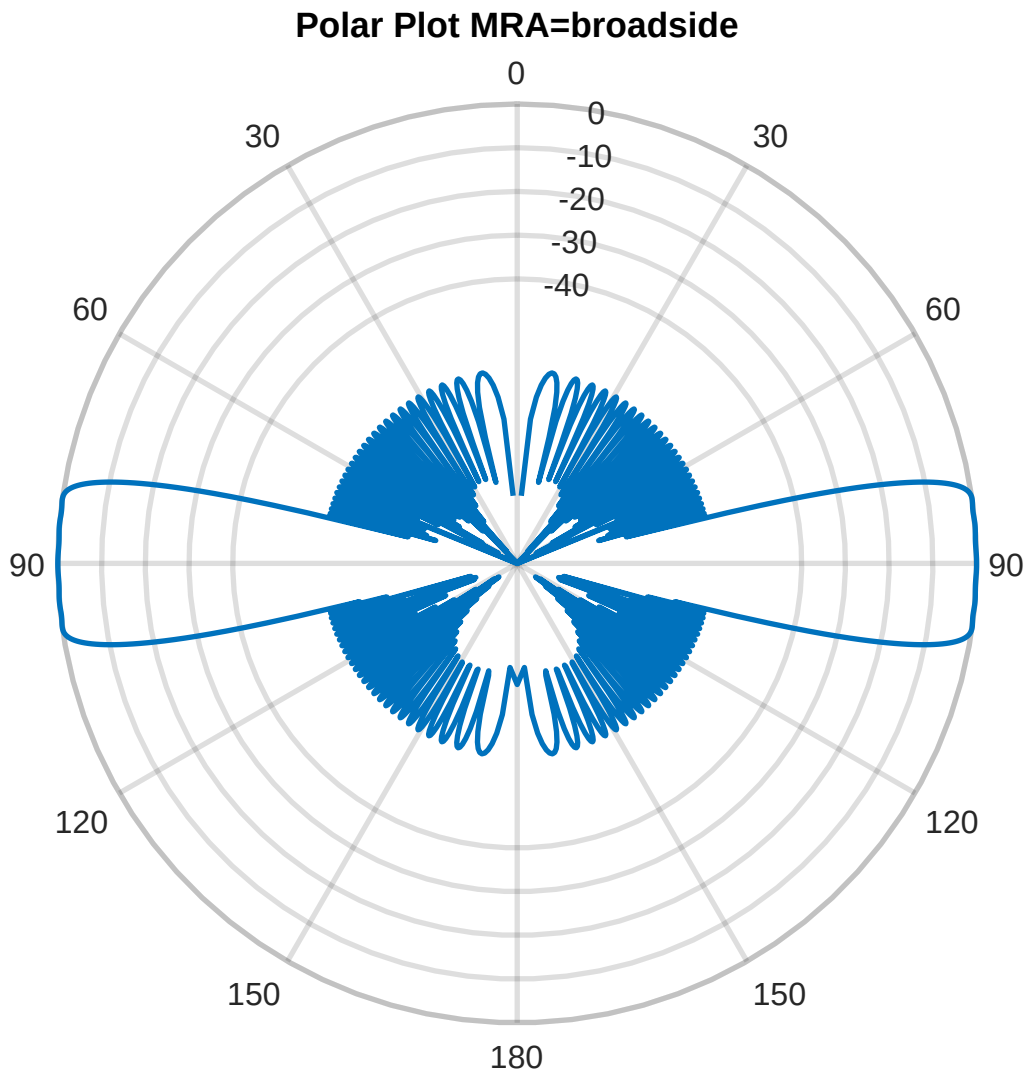
```
%% plots
figure
plot([0 wp ws 1],[1 1 0 0],w/pi,abs(h))
title('filter response')
grid on

figure
plot(psi/pi, B_psi_db, 'LineWidth', 2)
title('\psi plot')
grid on

figure
polarplot([theta 2*pi-flip(theta)], [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Polar Plot MRA=broadside')
ax = gca;
rlim([min(B_theta_db) max(B_theta_db)])
ax.RTick = -40:10:0;
ax.ThetaZeroLocation = 'top';
ax.ThetaDir = 'clockwise';
ax.ThetaTickLabel = {'0','30','60','90', '120', '150', '180', '150', '120',
'90', '60', '30'};
ax.FontSize = 12;
ax.LineWidth = 2;
grid on

figure
plot([theta 2*pi-flip(theta)]*180/pi, [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Transition bandwidth = 5 degree')
grid on

%%  Steer Angle = 60 degree
psi_t = 2*pi/3; % = 2*pi*(d/lambda)*cos60
[psi_mat, arg_mat] = meshgrid(psi-psi_t, -(N-1)/2:(N-1)/2);
v_psi = exp(1i*psi_mat.*arg_mat);
B_psi = conj(wts)*v_psi/sum(wts);
B_psi_db = 10*log10(abs(B_psi).^2);
B_theta_db = flip(B_psi_db);

figure
plot(psi/pi, B_psi_db, 'LineWidth', 2)
title('\psi plot (Steered at 60 degree)')
grid on

figure
```

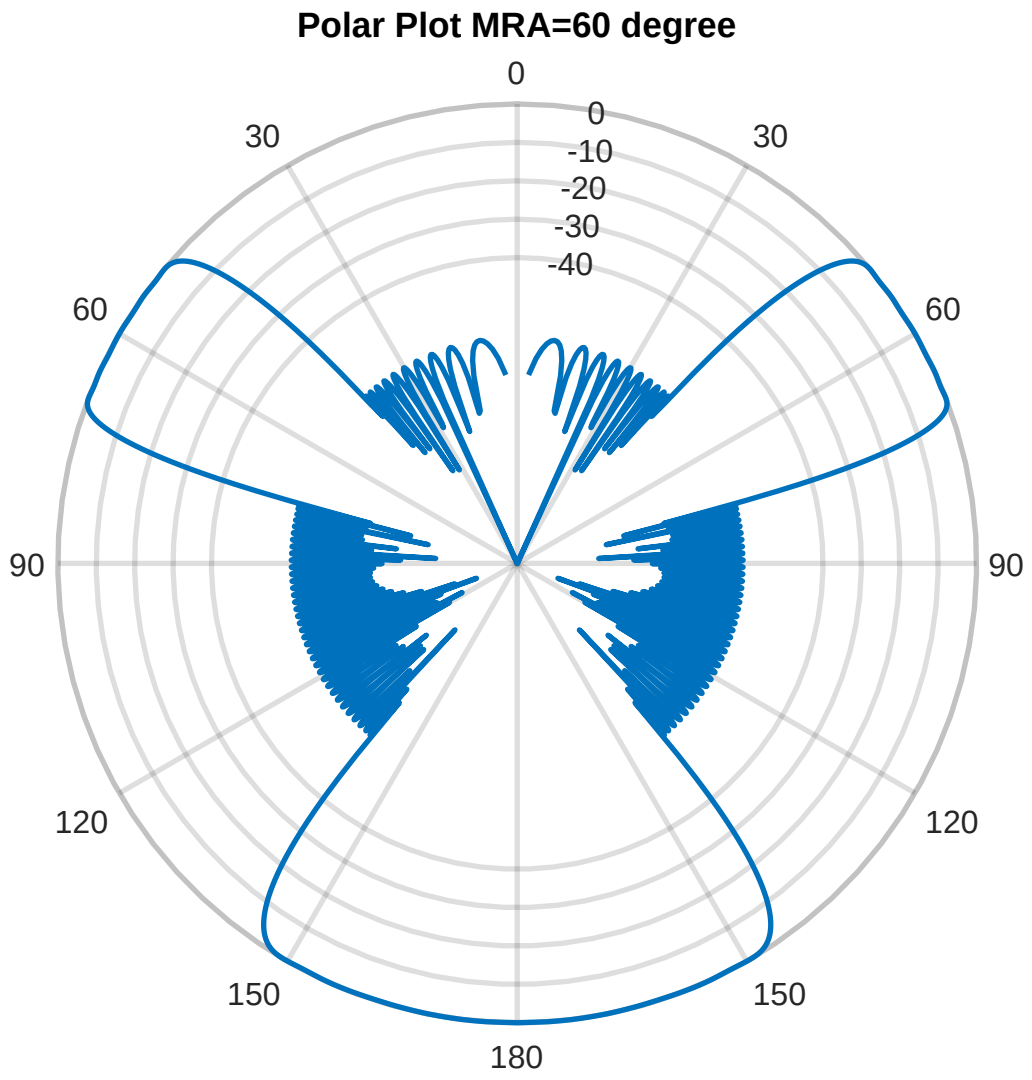
```
polarplot([theta 2*pi-flip(theta)], [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Polar Plot MRA=60 degree')
ax = gca;
rlim([min(B_theta_db) max(B_theta_db)])
ax.RTick = -40:10:0;
ax.ThetaZeroLocation = 'top';
ax.ThetaDir = 'clockwise';
ax.ThetaTickLabel = {'0','30','60','90', '120', '150', '180', '150', '120',
'90', '60', '30'};
ax.FontSize = 12;
ax.LineWidth = 2;
grid on

figure
plot([theta 2*pi-flip(theta)]*180/pi, [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Beamwidth = 20 degree')
grid on
```


Passband ripple
0.01
0.005
0
-0.005
-0.01
E(ψ)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
ψ/π
Stopband ripple
$\times 10^{-3}$
1
0.8
0.6
0.4
0.2
0
E(ψ)
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ψ/π


Transition bandwidth = 5 degree (in θ-space)
passband edge = 98.6
stopband edge = 103.6
0
-20
-40
-60
-80
B(θ) in dB
95
100
105
110
115
120
125
θ (in degrees)


Beamwidth = 20 degree (when steered to 60 degree)
0
-10
-20
-30
-40
-50
-60
-70
-80
B(θ) in dB
θ = 49.49
θ = 69.49
30
40
50
60
70
80
θ (in degrees)


Polar Plot MRA=broadside
0
30
60
90
120
150
180
0
-10
-20
-30
-40


Polar Plot MRA=60 degree
0
30
30
60
60
90
90
120
120
150
150
180
0
-10
-20
-30
-40