Array Design

The chosen design parameters are $N = 48$ and $\frac{d}{\lambda} = \frac{2}{3}$

**Explanations:** We satisfy the given constraints as follows:
1. The array is to be used for steering towards broadside over the range $(60^\circ, 120^\circ)$. Thus,
   \[ \frac{d}{\lambda} \leq \frac{1}{1 + \cos 60^\circ} = \frac{2}{3} \]

3. The BF has a beamwidth of $20^\circ$ when steered to $60^\circ$. Use the passband edges to determine the beamwidth.

Let the passband edge be at $\psi_p$. Then, we have

\[ \psi_t - \psi_p = 2\pi \frac{d}{\lambda} \cos B \quad (1) \]
\[ \psi_t + \psi_p = 2\pi \frac{d}{\lambda} \cos b \quad (2) \]

where, $\psi_t = 2\pi \frac{d}{\lambda} \cos 60^\circ$ and $B$ and $b$ are beam edges in theta-space when the beam is steered to $60^\circ$. Since beamwidth is $20^\circ$, we have

\[ B - b = \frac{\pi}{9} = \cos^{-1} \left\{ \frac{\lambda}{2\pi d}(\psi_t - \psi_p) \right\} - \cos^{-1} \left\{ \frac{\lambda}{2\pi d}(\psi_t + \psi_p) \right\} \quad (3) \]

We solve the above equation for the passband edge, $\psi_p$

2. The BF former has a low pass filter shape when steered towards broadside with a passband ripple of .01 and stopband ripple of .001. The transition bandwidth is $5^\circ$. Use Parks-McClellan approach for filter design

We shall use the above information to find the stopband edge $\psi_s$. When steered to broadside i.e $\psi_t = 0^\circ$, let $x$ be the corresponding passband edge in theta-space, measured from broadside. Thus,

\[ \psi_p = 2\pi \frac{d}{\lambda} \cos(90^\circ - x) \quad (4) \]
\[ \psi_p \frac{\lambda}{2\pi d} = \sin x \quad (5) \]
\[ x = \sin^{-1} \left\{ \psi_p \frac{\lambda}{2\pi d} \right\} \quad (6) \]

The stopband edge is then given by,

\[ \psi_s = 2\pi \frac{d}{\lambda} \cos(90^\circ - x - 5^\circ) \quad (7) \]

Using, $\psi_p$ and $\psi_s$ we then design the Parks-McClellan filter. The number of array elements is chosen so as to constrain the ripples in passband to 0.01 and
stopband to 0.001. This phenomena starts happening for \( N \geq 48 \). Thus, we chose \( N = 48 \).

**Code:**

```matlab
% Homework 2: Design Problem
close all

%% params

% Fixed
lambda = 2/3;
psi = -2*pi*lambda:0.01:2*pi*lambda;
theta = flip(acos(3*pi/(4*pi)));

%wp = 0.1;
%ws = 0.2;

% Solving subpart 3 to find wp
syms x
% to ensure beamwidth=20 degree when steer=60 degree
solu = vpasolve(acos(0.5-x*3/(4*pi))-acos(0.5+x*3/(4*pi)) == pi/9, x);
wp = double(solu)/pi;

% Using wp and subpart 2 to find ws
wp_thet = asin(pi*wp*3/(4*pi));
ws = 4*sin(wp_thet+5*pi/180)/3;

% Design params
N = 48;
ford = N-1; %filter order

%% Compute final beampattern
b = firpm(ford,[0 wp ws 1], [1 1 0 0], [1 10]);
freqz(b);

%wts = ifft(h);
wts = b;
[psi_mat, arg_mat] = meshgrid(psi, -(N-1)/2:(N-1)/2);
v_psi = exp(1i*psi_mat.*arg_mat);
B_psi = conj(wts)*v_psi/sum(wts);
B_psi_db = 10*log10(abs(B_psi).^2);
B_theta_db = flip(B_psi_db);

%% Filter response in Passband and Stopband
pb_ripple = max(abs(abs(h(w<=wp*pi)) - 1))
sb_ripple = max(abs(h(w>=ws*pi)))
```

2
%% plots
figure
plot([0 wp ws 1, [1 1 0 0], w/pi, abs(h))
title('filter response')
grid on

figure
plot(psi/pi, B_psi_db, 'LineWidth', 2)
title('psi plot')
grid on

figure
polarplot([theta 2*pi-flip(theta)], [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Polar Plot MRA-broadside')
ax = gca;
rlim([min(B theta_db) max(B theta_db)])
ax.RTick = -40:10:0;
ax.ThetaZeroLocation = 'top';
ax.ThetaDir = 'clockwise';
ax.ThetaTickLabel = {'0', '30', '60', '90', '120', '150', '180', '150', '120', '90', '60', '30'};
ax.FontSize = 12;
ax.LineWidth = 2;
grid on

figure
plot([theta 2*pi-flip(theta)]*180/pi, [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Transition bandwidth = 5 degree')
grid on

%% Steer Angle = 60 degree
psi_t = 2*pi/3; % = 2*pi*(d/lambda)*cos60
[psi_mat, arg_mat] = meshgrid(psi - psi_t, -(N-1)/2:(N-1)/2);
v_psi = exp(1i*psi_mat.*arg_mat);
B_psi = conj(wts)*v_psi/sum(wts);
B_psi_db = 10*log10(abs(B_psi).^2);
B theta_db = flip(B psi_db);

figure
plot(psi/pi, B_psi_db, 'LineWidth', 2)
title('psi plot (Steered at 60 degree)')
grid on

figure
polarplot([theta 2*pi - flip(theta)], [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Polar Plot MRA=60 degree')
ax = gca;
rlim([min(B_theta_db) max(B_theta_db)])
ax.RTick = -40:10:0;
ax.ThetaZeroLocation = 'top';
ax.ThetaDir = 'clockwise';
ax.ThetaTickLabel = {'0','30','60','90', '120', '150', '180', '150', '120', '90', '60', '30'};
ax.FontSize = 12;
ax.LineWidth = 2;
grid on

figure
plot([theta 2*pi - flip(theta)]*180/pi, [B_theta_db fliplr(B_theta_db)],
'LineWidth', 2)
title('Beamwidth = 20 degree')
grid on
Transition bandwidth = 5 degree (in $\theta$-space)

Passband edge = 98.6

Stopband edge = 103.6
Beamwidth = 20 degree (when steered to 60 degree)
Polar Plot MRA=broadside
Polar Plot MRA=60 degree