Computer Assignment

Using the spectral factorization method and the park-McClellan method, design a 2-channel perfect reconstruction (PR) filter bank with filter order 35. Please experimentally document that your design is PR.

Solution

**Step 1 - Design a half-band filter \( g \) of order \( 2N \) using Parks-McClellan method**

We can choose \( \omega_p = 0.44\pi \) and \( \omega_s = 0.56\pi \) (other choices such that \( \omega_p + \omega_s = \pi \) are also applicable). \( N=35 \) is as given in the problem. We also have to add the ripple term in the end to make the frequency response of the filter always positive (we add twice the maximum ripple value for safety). The Matlab code for this part is as:

```matlab
% Design a half-band filter of order 2N using Parks-McClellan Method
N = 35;
wp = 0.44;
ws = 0.56;
g_freq = [0.0 wp ws 1.0];
g_mag = [1.0 1.0 0.0 0.0];
[g err] = firpm(2*N,g_freq,g_mag);
g(N+1) = g(N+1)+2*err;
```
**Step 2: Perform spectral factorization**

Spectral factorization can be done by assigning all the zeros inside the unit circle and half of the zeros on the unit circle of the half-band filter $g$ we just designed to $H_0$. This can be done by first finding out all the zeros of $g$, sorting them in ascending order, and then assigning the first half of them to $H_0$. The following code implements these steps:

```matlab
% Perform spectral factorization
% Assign appropriate subset of zeros to h0 (all the zeros inside the unit
% circle and half of the zeros on the unit circle are being assigned)
g_roots = sort(roots(g));
h0 = poly(g_roots(1:N));
```

However, due to some finite precision effects of the filter design software, the filter $H_0$ we just obtained might not be as ideal as we had expected. Checking the coefficients of $H_0(z)H_0(z^{-1})$ we will find that the coefficients corresponding to the even power of $z$, except for 0, are not exactly zero, but some very small values. To deal with this problem, we first zero out these small values and then perform spectral factorization again to get the correct $H_0$. This is done by the following code:

```matlab
% Deal with finite precision effects
H_z = tf(h0,1,[],'variable','z^-1');
H_invz = tf(flip(h0),1,[],'variable','z');
Hf = H_z*H_invz;
Hf = cell2mat(Hf.num);
z0 = Hf(36);
Hf(2:2:end) = 0;
Hf(36) = z0;
Hf_roots = sort(roots(Hf));
h0 = poly(Hf_roots(1:N));
```
Step 3: Find $H_1$, $F_0$, and $F_1$

These filters can be derived from $H_0$ using the following relations:

\[
H_1(z) = z^N H_0(-z^{-1}) \\
F_0(z) = z^N H_0(z^{-1}) \\
F_1(z) = z^{-N} H_1(z^{-1})
\]

Or to be more convenient for our time-domain design consideration, we use:

\[
h_1[n] = (-1)^n h_0[N-n] \\
f_0[n] = h_0[N-n] \\
f_1[n] = h_1[N-n]
\]

The following code finds these filters:

```matlab
% Find h1, g0, g1 from h0
h1 = flip(h0);
for i=1:length(h1)
    h1(i) = h1(i)*(-1)^(i-1);
end
f0 = flip(h0);
f1 = flip(h1);
```

Step 4: Check if our system gives perfect reconstruction (PR)

We know that the output of a PR system is nothing but the time-shifted and scaled version of the input. We can apply an impulse signal to our system and see if the output is a time-shifted impulse. If it is, then we can show that our design is PR. In addition, we can also verify PR by checking if the sum: $|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 = constant, \forall \omega$.

The following code conducts these tests and proves the perfect reconstruction.
% Check if PR
input = zeros(1,100);
input(1) = 1;
v0 = downsample(conv(input,h0),2);
v1 = downsample(conv(input,h1),2);
y1 = conv(upsample(v0,2),f0);
y2 = conv(upsample(v1,2),f1);
output = y1+y2;
figure
stem(output)
title('Impulse response of the 2-channel filter')
xlabel('n')

[H0,w] = freqz(h0);
[H1,w] = freqz(h1);
n = 1:length(H0);
figure
plot((n-1)/length(H0),abs(H0).^2+abs(H1).^2);
title('|H_0(e^{j\omega})|^2+|H_1(e^{j\omega})|^2')
xlabel('Normalized Frequency (x rad/sample)')
\[ |H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 \]
The 4 filters and their frequency responses are also shown as follows:

![Graphs showing filters and frequency responses](image-url)
$F_0(e^{j\omega})$ and $F_1(e^{j\omega})$