Opportunistic Channel-Aware Spectrum Access for Cognitive Radio Networks with Interleaved Transmission and Sensing

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Abstract

Opportunistic spectrum access in a cognitive radio network has been a challenge due to the dynamic nature of spectrum availability and possible collisions between the primary user (PU) and the secondary user (SU). To maximize the spectrum utilization, we propose a spectrum access strategy where SU's packets are interleaved with periodic sensing to detect PU's return. Similar to earlier works on distributed opportunistic scheduling (DOS), we formulate the sensing/probing/access process as a maximum rate-of-return problem in the optimal stopping theory framework and show that the optimal channel access strategy is a pure threshold policy. We consider a realistic channel and system model by taking into account channel fading and sensing errors. We jointly optimize the rate threshold and the packet transmission time to maximize the average throughput of SU, while limiting interference to PU. Our numerical results show that significant throughput gains can be achieved with the proposed scheme compared to other well-known schemes. Our work sheds light on designing DOS protocols for cognitive radio with optimal transmission time that takes into account the dynamic nature of PUs.

Index Terms

Cognitive Radio Networks, Opportunistic Spectrum Access, Optimal Stopping, Periodic Sensing, Sensing Errors, Transmission Time Optimization

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I. Introduction

A. Motivation

The ever-increasing demand for higher spectrum efficiency in wireless communications due to limited or under-utilized spectral resources has infused a great interest in finding techniques for improving the spectrum usage. Cognitive radio appears as one very viable technology that can optimize the use of available radio frequency spectrum [1]. The concept of cognitive radio allows secondary users (SUs) to reuse spectral white spaces of primary users (PUs) in an opportunistic manner, without causing harmful interference to PUs [2].

It is essential for SU to make good sensing decisions in real-time to explore and utilize such opportunities for data transmission [3]. The conventional approaches for SU to access channels mainly focus on sensing the channels and transmitting on the ones that are deemed idle regardless of channel quality [3]. Recent results in [4], [5] show that by taking the channel conditions into account, in addition to the idle/busy status, the network throughput can be improved.

B. Main Contributions

In this paper, we propose an optimal spectrum access strategy involving transmission interleaved with periodic sensing that leverages sensing, channel-aware scheduling and optimization of transmission time in a joint manner to maximize SU's throughput. One of the key observations on cognitive radio is that the successful transmission of SU depends on PUs' activities. The return of PU would cause the transmission of SU to fail. However, while SU is transmitting, it has no knowledge of the return of PU. We therefore propose periodic sensing while transmission to track PU. In channel-aware scheduling and transmission with periodic sensing, there are two stages [6]. First, channel sensing is carried out to explore a spectrum hole for SU's transmission. Second, while a channel is used by SU, periodic sensing is deployed to detect the return of PU. The benefit of periodic sensing is that when PU returns, only the data transmitted since the last successful sensing may be lost – prior transmitted packets are not affected.

In transmission with periodic sensing, there exists a tradeoff between data lost due to PU's return using long packets, and the time cost of frequent sensing using short packets. If the transmission time is long, i.e., the frequency of periodic sensing is low, the time cost of tracking the return of PU is small but the amount of lost data when PU returns is large. On the contrary, if the transmission time is small and the frequency of periodic sensing is high, the amount of lost data when PU returns is small but at the expense of high cost of tracking PU. Motivated by this, we optimize the transmission time of SU between consecutive sensing phases to maximize the network throughput, which is equivalent to optimizing the frequency of periodic sensing.

We consider a system consisting of multiple channels. For channel searching, we adopt sequential channel scanning without recall [7] since SU may not be able to sense many channels at once due to the limitation on hardware and/or sensing capability. We characterize the joint sensing, probing and channel access with optimal transmission duration in a stochastic decision-making framework and formulate the decision problem as an optimal stopping problem [8]. When the sensing indicates that a channel is idle, probing is carried out to estimate the channel quality and the highest data rate it can support. Based on this estimate, one can decide either to proceed with transmission on this channel or to give up the opportunity and continue sensing for a potentially better channel. Clearly, further sensing/probing increases the likelihood of finding an idle channel with better rate, but at the cost of additional time. We show that the optimal channel access strategy exhibits a threshold structure, i.e., the channel access decision can be made by comparing the rate to a threshold. Furthermore, we jointly optimize the threshold and the transmission time between consecutive sensing phases to maximize the average throughput. This is done by alternately optimizing the threshold while keeping transmission time fixed using fixed-point iterations similar to [8], and followed by optimizing the transmission time keeping the threshold fixed using Newton's method.

In a practical cognitive radio network, spectrum sensing is not always accurate due to feedback delays, estimation errors and quantization errors. We say that a misdetection occurs if a channel is being used by PU but is incorrectly determined to be idle by SU. On the other hand, a false alarm happens if SU incorrectly determines that a channel is busy when in fact it is idle. Both situations are caused by sensing errors, which leads to the degradation of spectrum efficiency. In this paper, we take the sensing errors into account and determine their impact on our proposed channel access scheme. Given a certain probability of misdetection and false alarm, we determine the optimal transmission time and threshold under the sensing errors.

C. Related Works

The emergence of cognitive radio technology has stimulated a flurry of research activities in the area of dynamic spectrum access. We highlight some of the related channel access schemes.

Motivated by the rich channel diversity inherent in wireless communications, channel knowledge can be used as one criterion for channel selection to improve spectrum efficiency in wireless networks [4], [5], [8], [9]. Zheng *et al.* [8] use optimal stopping theory to develop distributed opportunistic scheduling (DOS) for exploiting multiuser diversity and time diversity in a single channel model for wireless *ad hoc* networks. Chang *et al.* [9] address the optimal channel selection problem in a multichannel system by considering the channel conditions. In our work, besides gaining the benefits of channel knowledge, we consider cognitive radio networks with incumbent PUs and also optimize the transmission time of SU to maximize throughput.

Shu *et al.* [4] show that joint channel sensing/probing scheme for cognitive radio can achieve significant throughput gains over conventional mechanisms that use sensing alone. They consider multiple channels and the throughput maximizing decision strategy is formulated as an optimal stopping theory problem. Our channel access scheme is an extension of optimal stopping results in [8], [4] and is more complex due to the variable transmission times, probing of the channels only when they are sensed to be idle and consideration of sensing errors. Additionally, we consider periodic sensing while transmission to track the return of PU and minimize collision. We further optimize the transmission time, i.e. the frequency of sensing. This helps us to efficiently utilize the idle state of channels that are explored.

There are few works in the literature that explicitly optimize the transmission time or perform periodic sensing while transmission [10], [11], [5]. Pei *et al.* [10] optimize the frame duration to maximize the throughput of the cognitive radio network subject to a fixed sensing time. They address the tradeoff that larger frame sizes allow for higher fraction of transmission time, but at a higher risk of collision and frame loss when the PU returns. They consider a slotted single channel – the SU does not transmit in a frame when an active PU is detected and waits until the next frame. In contrast, our work considers multichannel unslotted system and also takes channel quality into consideration before accessing an idle channel. Huang *et al.* [11] consider a model consisting of a single channel with a PU and SU and develop a scheme where the SU decides to transmit a packet or sense the channel based on its instantaneous estimate of

PU's idle probability under a POMDP framework. The SU may therefore transmit multiple consecutive packets after sensing the channel each time and can be considered as optimizing the transmission time between the sensing phases. They do not utilize channel quality information in their scheme. Li *et al.* [5] consider a scheme that is closely related to ours in a multichannel *ad hoc* network. They consider a model where the channel quality gradually changes with time and therefore monitor the channel quality periodically while transmission, stopping when the quality falls below a threshold. But the packet transmission time is fixed and not optimized.

Apart from the optimal stopping theory approach to the channel access problem, another popular approach in the literature is based on the POMDP framework. Zhao *et al.* [3] and Chen *et al.* [12], [13] study such spectrum access schemes for slotted multichannel cognitive radio networks. POMDP-based schemes attempt to dynamically track the idle state of various channels and maximize throughput by exploiting the temporal spectrum opportunities. Like most schemes based on optimal stopping theory, our scheme explores channels uniformly at random and doesn't dynamically track the idle channels, but it does fully utilize the idle state of the channels it accesses. As we show in the numerical results, our scheme that is unslotted, takes channel quality into consideration, performs periodic sensing while transmission and jointly optimizes the packet duration and the channel quality threshold, outperforms the POMDP scheme in [3] for slotted systems. Zhao *et al.* [14] study the dynamic access using a periodic sensing strategy under a constrained Markov decision process framework. However, they consider the packet transmission time to be given and fixed, and do not optimize it.

Most recent works relate to a wide variety of other important concerns like energy-efficient transmission for cognitive radio sensor networks, *e.g.,* [15], [16], game theoretic and security considerations in non-cooperative multiuser setups, *e.g.,* [17], [18], and machine learning approaches when the various channel parameters and related probability distributions are not known, *e.g.,* [19], [20] and the references therein. While a treatment of periodic sensing and joint optimization packet duration and channel quality threshold in such contexts is beyond the scope of this paper, we believe that these concepts can be used in conjunction with existing schemes and provide significant performance benefits.

The remainder of the paper is organized as follows. We present the channel and system model in Section II. In Section III, we present the throughput-optimal channel access strategy and in

Section IV, we provide the average throughput analysis. We then present the joint optimization of threshold and transmission duration in Section V. In Section VI, we consider interference of SU to PUs and in Section VII, we consider extensions of our scheme to more general scenarios. We present the numerical results in Section VIII. Finally, we conclude the paper with Section IX.

II. Channel and System Model

A. Channel Model

We consider a frequency-selective multi-channel system such as orthogonal frequency-division multiple access (OFDMA) that is commonly used for cognitive radios, e.g., the IEEE 802.22 [21] wireless standard. The entire frequency spectrum is assumed to be divided into L independent and identically distributed (*i.i.d.*) channels. We assume that the coherence bandwidth is bigger than the signal bandwidth of the individual channels or the subcarriers in it, and thus each channel experiences flat fading. Furthermore, we assume that each channel experiences slow fading, i.e., its condition varies slowly over time.

We further assume that all channels have the same statistics, and are subject to Rayleigh fading. While the homogeneous setup is assumed for simplicity and may correspond to the case when channels belong to the same network, we consider extensions to the heterogeneous case in subsection VII-A. The distribution of rate R is continuous and is given by the Shannon channel capacity $R = \log(1 + \rho|h|^2)$ nats/s/Hz, where ρ is the normalized average SNR, and h is the random channel coefficient with a complex Gaussian distribution $\mathcal{CN}(0, 1)$. Accordingly, the distribution of the rate is given by

$$F_R(r) = 1 - \exp\Big(-\frac{\exp(r) - 1}{\rho}\Big) \tag{1}$$

for $r \geq 0$, and $F_R(r) = 0$ otherwise.

B. System Model

1) PU and SU Model: We assume that each channel has only one designated PU. The L channels are opportunistically available to SU. Although we consider a system where there is only one SU, it can be readily extended to the case when there are multiple SUs, as discussed later in Subsection VII-B.

Each channel's status is modeled as a continuous-time random process that alternates between busy and idle states depending on whether PU is using the channel. Specifically, we consider a system in which the idle/busy states of different PU channels are homogeneous, independent and identically distributed. This is motivated by the common scenario that all the L channels belong to the same primary licensed network [4] and may therefore have similar usage statistics. We assume that for all PUs, the time durations of the idle and busy states are exponentially distributed with parameters a and b [22]. In other words, for any PU, the duration T_I of any idle state has distribution $f_{T_I}(t) = ae^{-at}$ and the duration T_B of any busy state has distribution $f_{T_B}(t) = be^{-bt}$. The expected durations of each of the idle and busy states are $\frac{1}{a}$ and $\frac{1}{b}$ respectively. The fraction of time for which PU is idle in the long term is the idle probability $P_{\mathrm{I}} = \frac{1/a}{1/a+1/b} = \frac{b}{a+b}$.

2) Channel Sensing, Probing and Data Transmission: For selecting a channel, SU uses the scheme of sequential sensing and probing without recall [7]. Here, SU senses/probes the channels sequentially and does not have the memory of the previously sensed/probed channels and their outcomes. Therefore, SU cannot recall or select a previously sensed/probed channel once it forgoes the opportunity to transmit on that channel, unless the sensing/probing is repeated on that channel.

To obtain a better sense of the dynamics of channel access, a sample realization of the sensing/probing for channel selection followed by data transmission on that channel with periodic sensing is depicted in Fig. 1. When SU intends to transmit, it searches for an available channel by randomly choosing channels one at a time and sensing/probing them. The total time spent for channel searching depends on the activities of PUs and the channel conditions. Specifically, if the outcome of the sensing stage is busy, the probing stage is skipped and SU randomly selects another channel for sensing. In this case, the time spent for sensing/probing a busy channel is τ_{s}. However, if the sensed channel is idle, SU proceeds with probing to determine the channel quality for deciding whether to transmit on the channel. During the probing stage, a channel probing packet (CPP) and a probing feedback packet (PFP) are exchanged between the transmitter and receiver [4]. The time spent on a CPP/PFP exchange is denoted by the channel probing time τ_{p} and in this case, the time cost for sensing/probing an idle channel is $\tau_{\mathrm{s}} + \tau_{\mathrm{p}}$. With the feedback information on the channel quality, the transmitter compares the maximum achievable data rate to an optimal threshold (λ^*) pre-designed using the optimal stopping theory. If the data rate is less than the threshold due to poor channel condition, then SU forgoes its transmission opportunity and continues with sensing/probing another randomly selected channel. However, if the data rate is high and exceeds the threshold, then SU proceeds with the data transmission.

During SU's data transmission, it has no knowledge of the return of PU. If PU returns during the transmission of a SU's packet, then that entire packet is lost. Hence, to maximize the chances of successful transmission of SU's packets and to reduce the interference of SU to PU, it is necessary for SU to track the activity of PU. We therefore propose periodic sensing during the transmission. Specifically, SU will periodically sense the channel after transmitting for time $T_{\rm s}$. Note that $T_{\rm s}$ is the duration of SU's transmission between two consecutive sensing phases and is also equivalent to the length of a sub-packet of SU. The transmission of SU stops once it senses the return of a PU during a sensing phase. If PU returns during SU's transmission, then interference occurs and the current sub-packet being transmitted is destroyed, but the previously transmitted sub-packets are still valid. During SU's transmission, only sensing is performed periodically, but not the probing. This is because the channel condition is assumed to be constant over a long period of time. The transmitter and the receiver are assumed to be synchronized. Under the same spectrum access strategy, the transmitter and the receiver will always sense, probe and access the same channel.

3) Spectrum Sensing Model: Spectrum sensing can be modeled as hypothesis testing. It is equivalent to distinguishing between the two hypotheses:

$$\begin{cases} H_0: y(t) = n(t), & \text{PU is inactive} \\ H_1: y(t) = x(t) + n(t), & \text{PU is active} \end{cases} \tag{2}$$

where $x(t)$ denotes PU's transmitted signal, $n(t)$ is additive white Gaussian noise and $y(t)$ denotes sample collected by SU. The notation H_0 represents the hypothesis that PU is inactive (idle channel) whereas H_1 indicates that PU is active (busy channel).

In a practical system, there may be sensing errors and accordingly, we define the probability of false alarm $P_{\rm fa}$ and miss detection $P_{\rm md}$ as

$$P_{\rm fa} = \Pr(I = 0|H_0) \quad \text{and} \quad P_{\rm md} = \Pr(I = 1|H_1), \tag{3}$$

where $I = 0$ indicates that SU decides the channel is busy and $I = 1$ indicates idle.

III. DERIVATION OF THROUGHPUT-OPTIMAL CHANNEL ACCESS STRATEGY

We consider the problem of finding an optimal strategy for SU to decide whether or not to transmit on an idle channel based on its quality, so as to maximize the long-term average throughput. We show that for any given packet length $T_{\rm s}$, an optimal strategy for the SU is to select the first idle channel whose rate exceeds a fixed threshold $\lambda^* \triangleq \lambda^*(T_{\rm s})$. For this, we consider a maximum rate-of-return problem in the optimal stopping theory framework [23], [24]. An optimal stopping rule is a strategy to decide as to when one should take a given action based on the past events in order to maximize the average return. The return is defined as the net gain between the reward achieved and the cost spent. In our problem, the reward is the rate of the channel probed and the cost is the total time taken to explore the channels so far.

As illustrated in Fig. 1, after finding an idle channel, a stopping rule N decides whether SU should carry out the data transmission, or skip this transmission opportunity, based on the channel quality. As such, N is the number of idle channels considered by SU before deciding to transmit on the last idle channel based on the channel qualities and the time spent so far. One can see that further sensing/probing would certainly increase the probability of getting an available channel with a better channel quality, but at the expense of spending additional time in searching. Using the optimal stopping theory, this tradeoff can be characterized in a stochastic decision making framework.

Suppose that the process of successful sensing/probing followed by transmission is carried out for U rounds. Let $\{N_1, \ldots, N_U\}$ be the corresponding number of idle channels considered in these rounds, and are independent realizations of N. Let T_{N_u} denote the total duration of round u which includes sensing/probing with transmission and periodic sensing. And let R_{N_u} be the data rate of the channel used in round u. Based on the Renewal Theorem [8], the average throughput after U rounds is given by

$$x_U \triangleq \frac{\sum_{u=1}^{U} R_{N_u} T'_u}{\sum_{u=1}^{U} T_{N_u}} \overset{U\to\infty}{\longrightarrow} \frac{E[R_N T']}{E[T_N]} \text{ a.s.} \tag{4}$$

Here, $x \triangleq \frac{E[R_N T']}{E[T_N]}$ is the long-term average rate-of-return for SU, T_N is the total duration of a round (i.e., time spent for channel searching and transmission), R_N is the transmission rate in a round and T' is the effective data transmission time in a round. Clearly, the distributions of R_N and T_N depend on the stopping rule N. The total time T_N of a round consists of the time

T'_N spent in sensing and probing to acquire a good channel and the time $T_{\rm tr}$ for transmitting SU's packets over this channel. The time $T_{\rm tr}$ includes both the successfully and unsuccessfully transmitted packets (due to PU's return and sensing errors) until SU senses PU's return, and the time spent due to periodic sensing between the packets. We have $\mathrm{E}[T_N] = \mathrm{E}[T'_N] + \mathrm{E}[T_{\rm tr}]$.

It follows that the problem of maximizing the long-term average throughput can be formulated as a maximal-rate-of-return problem [8]. Our goal is to find an optimal stopping rule N^* that maximizes the average rate-of-return x, and the corresponding maximal throughput x^*:

$$N^* \triangleq \arg\max_{N\in Q} \frac{E[R_N T']}{E[T_N]}, \quad x^* \triangleq \sup_{N\in Q} \frac{E[R_N T']}{E[T_N]}, \tag{5}$$

where $Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}$ is the set of all possible stopping rules. We exploit optimal stopping theory to solve (5).

Proposition 3.1. *There exists an optimal stopping rule N^* for the opportunistic spectrum access and is a pure threshold policy given by*

$$N^* = \min\big\{n \geq 1 : R_n \geq \lambda^*\big\}, \tag{6}$$

where the optimal threshold λ^* is the unique solution for λ in

$$\mathrm{E}\big[(R-\lambda)^+\big] = \frac{\lambda\big(\mathrm{E}[K_{\rm s}]\tau_{\rm s} + \mathrm{E}[K_{\rm p}]\tau_{\rm p}\big)}{\mathrm{E}[T_{\rm tr}]}. \tag{7}$$

Here, R is a r.v. which refers to the rate whose CDF is $F_R(r)$ shown in (1), and $K_{\rm s}$ and $K_{\rm p}$ are the number of channels sensed and probed respectively to find a channel in which PU is idle for the time $(\tau_{\rm s}+\tau_{\rm p})$. (Thus, $\mathrm{E}[K_{\rm s}]\tau_{\rm s}+\mathrm{E}[K_{\rm p}]\tau_{\rm p}$ is the expected time spent until SU finds an idle channel to probe completely.) Furthermore, the maximum throughput is given by $x^* = \frac{\lambda^*\mathrm{E}[T']}{\mathrm{E}[T_{\rm tr}]}$.

The proof can be found in Appendix A. Proposition 3.1 suggests that an optimal scheduling strategy has the following form: the successfully contended link will start the data transmission if the transmission rate from the probing is bigger than or equal to the λ^*. Else, the link will forgo the transmission opportunity.

IV. Throughput Analysis

To analyze the maximal throughput x^* and the optimal stopping rule N^*, we consider the calculation of x^* and λ^* in terms of the various channel and system model parameters. We first calculate the various expectations that were encountered in Proposition 3.1.

Proposition 4.1. *For any stopping rule that is a pure threshold policy* $N = \min\{n : R_n \geq \lambda\}$*, the expected times of effective transmission (*$\mathrm{E}[T']$*), channel access (*$\mathrm{E}[T'_N]$*), transmission with periodic sensing (*$\mathrm{E}[T_{\mathrm{tr}}]$*), and the rate of transmission (*$\mathrm{E}[R_N]$*) are given by*

$$\mathrm{E}[T'] = \frac{T_\mathrm{s} \cdot e^{-aT_\mathrm{s}}}{1 - e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1 - P_\mathrm{fa})},$$

$$\mathrm{E}[T'_N] = \frac{\tau_\mathrm{s} + Q'_\mathrm{I}\tau_\mathrm{p}}{(\frac{b}{a+b})e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1 - P_\mathrm{fa})(1 - F_R(\lambda))},$$

$$\mathrm{E}[T_\mathrm{tr}] \;=\; \frac{1 - P_\mathrm{md}e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}}{1 - P_\mathrm{md}} \frac{(T_\mathrm{s} + \tau_\mathrm{s})}{1 - e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1 - P_\mathrm{fa})},$$

$$\mathrm{E}[R_N] = \frac{\int_\lambda^\infty r \; dF_R(r)}{1 - F_R(\lambda)}.$$

Here, $Q'_\mathrm{I} = (\frac{a}{a+b})P_\mathrm{md} + (\frac{b}{a+b})\big((1 - e^{-a\tau_\mathrm{s}})P_\mathrm{md} + e^{-a\tau_\mathrm{s}}(1 - P_\mathrm{fa})\big)$ *is the probability of finding a channel in which PU is sensed to be idle. The expected number of channels sensed (*$\mathrm{E}[K_\mathrm{s}]$*) and probed (*$\mathrm{E}[K_\mathrm{p}]$*) for finding a channel in which PU is idle for the time* $(\tau_\mathrm{s} + \tau_\mathrm{p})$ *are given by*

$$\mathrm{E}[K_\mathrm{s}] = \frac{1}{(\frac{b}{a+b})e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1 - P_\mathrm{fa})} \quad \textit{and} \quad \mathrm{E}[K_\mathrm{p}] = \frac{Q'_\mathrm{I}}{(\frac{b}{a+b})e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1 - P_\mathrm{fa})}.$$

The expressions for $\mathrm{E}[T'], \mathrm{E}[T_\mathrm{tr}], \mathrm{E}[K_\mathrm{s}], \mathrm{E}[K_\mathrm{p}]$ *hold irrespective of the stopping rule being used.*

The proof mostly relies on properties of Poisson processes, and exponentially and geometrically distributed random variables, and is provided in Appendix B.

Using Proposition 4.1, it follows that for a threshold rule $N = \min\{n \geq 1 : R_n \geq \lambda\}$ with threshold λ, the rate of return in (4) is given by

$$x = \frac{E[R_N T']}{E[T_N]} = \frac{E[R_N]E[T']}{E[T'_N] + E[T_\mathrm{tr}]}$$
$$= \frac{\int_\lambda^\infty r \; dF_R(r) \cdot T_\mathrm{s}e^{-aT_\mathrm{s}}}{\frac{(\frac{a+b}{b})e^{a(\tau_\mathrm{s}+\tau_\mathrm{p})}(\tau_\mathrm{s}+Q'_\mathrm{I}\tau_\mathrm{p})(1-(1-P_\mathrm{fa})e^{-a(T_\mathrm{s}+\tau_\mathrm{s})})}{1-P_\mathrm{fa}} + \frac{(T_\mathrm{s}+\tau_\mathrm{s})(1-P_\mathrm{md}e^{-a(T_\mathrm{s}+\tau_\mathrm{s})})(1-F_R(\lambda))}{1-P_\mathrm{md}}} \triangleq \phi(\lambda, T_\mathrm{s}). \quad (8)$$

Using Proposition 3.1, since λ^* and x^* satisfy $x^* = \frac{\lambda^*\mathrm{E}[T']}{\mathrm{E}[T_\mathrm{tr}]}$, we have $\lambda^* = \frac{\mathrm{E}[T_\mathrm{tr}]}{\mathrm{E}[T']}x^* = \frac{\mathrm{E}[T_\mathrm{tr}]}{\mathrm{E}[T']}\phi(\lambda^*, T_\mathrm{s})$, i.e., λ^* is a solution to the fixed-point equation in λ, given by

$$\lambda \;=\; \frac{\mathrm{E}[T_\mathrm{tr}]}{\mathrm{E}[T']}\phi(\lambda, T_\mathrm{s}) \;=\; \frac{\mathrm{E}[R_N]}{\frac{\mathrm{E}[T'_N]}{\mathrm{E}[T_\mathrm{tr}]} + 1} \;=\; \frac{\int_\lambda^\infty r dF_R(r)}{c_0 - F_R(\lambda)} \;\triangleq\; \psi(\lambda). \quad (9)$$

Here, $c_0 = 1 + \frac{\mathrm{E}[T'_N](1-F_R(\lambda))}{\mathrm{E}[T_\mathrm{tr}]} = 1 + \frac{(\frac{a+b}{b})e^{a(\tau_\mathrm{s}+\tau_\mathrm{p})}(\tau_\mathrm{s}+Q'_\mathrm{I}\tau_\mathrm{p})(1-(1-P_\mathrm{fa})e^{-a(T_\mathrm{s}+\tau_\mathrm{s})})}{1-P_\mathrm{fa}} \frac{1-P_\mathrm{md}}{(T_\mathrm{s}+\tau_\mathrm{s})(1-P_\mathrm{md}e^{-a(T_\mathrm{s}+\tau_\mathrm{s})})}$ is a constant that does not depend on λ using Proposition 4.1.

Similar to [8, Prop. 3.4], we have the following result for finding λ^* when T_{s} is given.

Proposition 4.2. *For a given T_{s}, the fixed-point iteration*

$$\lambda_{k+1} = \psi(\lambda_k), \tag{10}$$

for $k \in \{0, 1, 2, \ldots\}$ and for any nonnegative λ_0 converges to the optimum threshold λ^*.

Proof is given in Appendix C.

V. Joint Optimization of Threshold and Transmission Duration

We jointly optimize the transmission time T_{s} and the threshold λ to maximize the throughput $\phi(\lambda, T_{\mathrm{s}})$ in (8). An illustration of the function $\phi(\lambda, T_{\mathrm{s}})$ is given in Fig. 2. We show that for a given threshold rule $N = \min\{n \geq 1 : R_n \geq \lambda\}$, i.e., for a fixed threshold λ, the optimum transmission time T_{s} that maximizes throughput can be obtained by taking the derivative with respect to T_{s} and equating to zero, i.e., solving for T_{s} in $\frac{\partial}{\partial T_{\mathrm{s}}}\phi(\lambda, T_{\mathrm{s}}) = 0$. To simplify this process, we express $\phi(\lambda, T_{\mathrm{s}})$ in (8) as

$$\phi(\lambda, T_{\mathrm{s}}) = \frac{c_1 T_{\mathrm{s}} e^{-aT_{\mathrm{s}}}}{c_2(1 - c_3 e^{-aT_{\mathrm{s}}}) + c_4(T_{\mathrm{s}} + \tau_{\mathrm{s}})(1 - c_5 e^{-aT_{\mathrm{s}}})} = \frac{c_1 T_{\mathrm{s}}}{c_4 T_{\mathrm{s}} e^{aT_{\mathrm{s}}} + c_6 e^{aT_{\mathrm{s}}} - c_7 T_{\mathrm{s}} - c_8}, \tag{11}$$

where

$$c_1 = \int_{\lambda}^{\infty} r \; dF_R(r), \quad c_2 = \frac{(\frac{a+b}{b}) e^{a(\tau_{\mathrm{s}}+\tau_{\mathrm{p}})}(\tau_{\mathrm{s}} + Q_{\mathrm{I}}' \tau_{\mathrm{p}})}{1 - P_{\mathrm{fa}}}, \quad c_3 = (1 - P_{\mathrm{fa}}) e^{-a\tau_{\mathrm{s}}},$$
$$c_4 = \frac{(1 - F_R(\lambda))}{1 - P_{\mathrm{md}}}, \quad c_5 = P_{\mathrm{md}} e^{-a\tau_{\mathrm{s}}}, \quad c_6 = c_2 + c_4\tau_{\mathrm{s}}, \quad c_7 = c_4 c_5, \quad c_8 = c_2 c_3 + c_4 c_5 \tau_{\mathrm{s}} \tag{12}$$

do not depend on T_{s}. We therefore have

$$\frac{\partial}{\partial T_{\mathrm{s}}}\phi(\lambda, T_{\mathrm{s}}) \;=\; \frac{c_6 e^{aT_{\mathrm{s}}} - c_8 - a c_4 T_{\mathrm{s}}^2 e^{aT_{\mathrm{s}}} - a c_6 T_{\mathrm{s}} e^{aT_{\mathrm{s}}}}{(c_4 T_{\mathrm{s}} e^{aT_{\mathrm{s}}} + c_6 e^{aT_{\mathrm{s}}} - c_7 T_{\mathrm{s}} - c_8)^2}. \tag{13}$$

We solve for T_{s} in $\frac{\partial}{\partial T_{\mathrm{s}}}\phi(\lambda, T_{\mathrm{s}}) = 0$, i.e., $c_6 e^{aT_{\mathrm{s}}} - c_8 - a c_4 T_{\mathrm{s}}^2 e^{aT_{\mathrm{s}}} - a c_6 T_{\mathrm{s}} e^{aT_{\mathrm{s}}} = 0$, i.e.,

$$\zeta(T_{\mathrm{s}}) \triangleq c_6 - c_8 e^{-aT_{\mathrm{s}}} - a c_6 T_{\mathrm{s}} - a c_4 T_{\mathrm{s}}^2 = 0. \tag{14}$$

The next proposition shows that the above equation has a unique solution T_{s}^* for $T_{\mathrm{s}} > 0$ and $T_{\mathrm{s}}^* < \frac{1}{a}$. Hence, T_{s}^* can be obtained by solving (14) using Newton's method with initial value $\frac{1}{a}$ and update equation or by bisection on the range $(0, \frac{1}{a})$.

Proposition 5.1. *For a threshold rule with given threshold* λ*, the optimal transmission time* T_{s}^* *that maximizes throughput* $\phi(\lambda, T_{\mathrm{s}})$ *is the unique solution to Equation (14). And* $T_{\mathrm{s}}^* \leq \frac{1}{a}$.

The proof is shown in Appendix D. Based on propositions 4.2 and 5.1, we propose Algorithm 1 for finding λ^* and T_{s}^* that jointly maximize the throughput $\phi(\lambda, T_{\mathrm{s}})$.

Algorithm 1 Joint optimization of λ and T_{s} to maximize throughput $\phi(\lambda, T_{\mathrm{s}})$

Given: sufficiently small error bounds ϵ_λ, $\epsilon_{T_{\mathrm{s}}}$
Initialize $\lambda = 1$, $T_{\mathrm{s}} = \frac{1}{a}$
repeat
    $\lambda^{\mathrm{old}} = \lambda$, $T_{\mathrm{s}}^{\mathrm{old}} = T_{\mathrm{s}}$
    repeat {Optimize λ for current T_{s} by fixed-point iterations}
        $\lambda = \psi(\lambda)$
    until $|\lambda - \psi(\lambda)| \leq \epsilon_\lambda / 2$
    repeat {Optimize T_{s} for current λ by Newton's method}
        $T_{\mathrm{s}} = \frac{c_8 e^{-aT_{\mathrm{s}}}(aT_{\mathrm{s}}+1) - ac_4 T_{\mathrm{s}}^2 - c_6}{a(c_8 e^{-aT_{\mathrm{s}}} - 2c_4 T_{\mathrm{s}} - c_6)} \left(= T_{\mathrm{s}} - \frac{\zeta(T_{\mathrm{s}})}{\frac{\partial}{\partial T_{\mathrm{s}}}\zeta(T_{\mathrm{s}})} \right)$
    until $|T_{\mathrm{s}} - \frac{c_8 e^{-aT_{\mathrm{s}}}(aT_{\mathrm{s}}+1) - ac_4 T_{\mathrm{s}}^2 - c_6}{a(c_8 e^{-aT_{\mathrm{s}}} - 2c_4 T_{\mathrm{s}} - c_6)}| \leq \epsilon_{T_{\mathrm{s}}}/2$
until $|\lambda^{\mathrm{old}} - \lambda| \leq \epsilon_\lambda$ and $|T_{\mathrm{s}}^{\mathrm{old}} - T_{\mathrm{s}}| \leq \epsilon_{T_{\mathrm{s}}}$
Return λ and T_{s} as approximations of λ^* and T_{s}^*

By propositions 4.2 and 5.1, the inner loops in the above algorithm converge to the best λ and T_{s} for the current T_{s} and λ respectively, and each inner loop leads to an increase in the rate $\phi(\lambda, T_{\mathrm{s}})$. Therefore, the algorithm converges to a local maximum of $\phi(\lambda, T_{\mathrm{s}})$. While propositions 4.2 and 5.1 show that $\phi(\lambda, T_{\mathrm{s}})$ has a unique maximum, i.e., is quasi-concave, in λ for a given T_{s} and has a unique maximum in T_{s} for a given λ, it does not guarantee that $\phi(\lambda, T_{\mathrm{s}})$ has a unique local maximum. For example, the function $g(x, y) = -x^4 + 6x^3 - 11y^2 + 6y$ has a unique maximum in x (resp. y) for a given y (resp. x), but has two local maxima, as seen from the fact that $g(x, x) = x(1 - x)(2 - x)(3 - x)$. Thus, the algorithm is not guaranteed to converge to the global maximum, except when $\phi(\lambda, T_{\mathrm{s}})$ has a unique local maximum. Based on our numerical results, *e.g.,* see Fig. 2, we strongly suspect that this is indeed true for rate distributions under Rayleigh fading.

While we do not have theoretical guarantees on the speed of convergence of Algorithm 1, in our experiments that we describe in Section VIII, the algorithm converges very fast, within 10 iterations, to within a small error of $\epsilon_{\lambda} = \epsilon_{T_{\rm s}} = 10^{-5}$.

Before we proceed to show numerical results for our scheme, in the next two sections, we consider modifications of our scheme to take into account the interference caused by SU to PUs and extensions to more general scenarios.

VI. Interference to Primary User

If PU returns during SU's transmission, there may be a collision, leading to interference to PU. The collision will continue until SU detects PU's presence in one of the following sensing phases. To minimize this interference, the transmission power of SU should be small or alternatively, the transmission time $T_{\rm s}$ of SU between two consecutive sensing phases should be small. One way of quantifying this interference is in terms of the fraction of time for which each PU experiences interference in the long term. We compute this fraction when the SU uses a threshold policy for channel access and periodic sensing while transmission, with corresponding threshold λ and packet length $T_{\rm s}$. Let $T_{\rm c}$ be the random time duration at the end of a round of transmission for which the SU experiences collision due to the return of the PU. In order to calculate the expected collision time in a round $\mathrm{E}[T_{\rm c}]$, we split $T_{\rm c}$ in two components. The first component $T_{\rm c,1}$ is due to PU returning between two sensing phases of SU, and is the time between the return of PU to the next sensing phase. The second is the additional collision time $T_{\rm c,2}$ if the SU continues to transmit due to misdetection in one or more of the sensing phases. To calculate $\mathrm{E}[T_{\rm c,1}]$, we observe that if the PU returns at time t from the start of SU's transmission, then the rest of SU's transmission causes collision, i.e., $T_{\rm c,1} = T_{\rm s} - t$. This happens with probability density ae^{-at} and we thus have $\mathrm{E}[T_{\rm c,1}] = \int_0^{T_{\rm s}} (T_{\rm s} - t) ae^{-at} dt = T_{\rm s} - \frac{(1-e^{-aT_{\rm s}})}{a}$.

To calculate $\mathrm{E}[T_{\rm c,2}]$, we see that for every misdetection, there is an additional collision time of $T_{\rm s}$. The number of such misdetections is a geometrically distributed random variable with expected value of $\frac{P_{\rm md}}{1-P_{\rm md}}$. Thus, $\mathrm{E}[T_{\rm c,2}] = T_{\rm s} \cdot \frac{P_{\rm md}}{1-P_{\rm md}}$, and

$$\mathrm{E}[T_{\rm c}] \;=\; \mathrm{E}[T_{\rm c,1}] + \mathrm{E}[T_{\rm c,2}] \;=\; T_{\rm s} - \frac{(1-e^{-aT_{\rm s}})}{a} + T_{\rm s} \cdot \frac{P_{\rm md}}{1-P_{\rm md}}. \quad (15)$$

Since the expected duration of a round is $\mathrm{E}[T_N]$, the fraction of time for which the SU experiences collision during transmission in the long term is $\frac{\mathrm{E}[T_{\rm c}]}{\mathrm{E}[T_N]}$. Since the SU is equally likely to transmit on each channel, the average fraction of time for which each PU experiences collision is

$$\eta_{\rm c} = \frac{1}{L} \cdot \frac{{\rm E}[T_{\rm c}]}{{\rm E}[T_N]}, \tag{16}$$

where ${\rm E}[T_{\rm c}]$ and ${\rm E}[T_N] = {\rm E}[T'_N] + {\rm E}[T_{\rm tr}]$ are given by Equation (15) and Proposition 4.1 respectively.

The collision $\eta_{\rm c}$ increases with $P_{\rm md}$ and high $P_{\rm md}$ may cause significant interference to PU. Hence, in a practical system, the requirement for $P_{\rm md}$ needs to be very small, i.e., $P_{\rm md} \ll 1$ [4].

We also see that $\eta_{\rm c}$ is an increasing function of $T_{\rm s}$. Given a bound $\hat{\eta_{\rm c}}$ on the interference, we consider the following modification of Algorithm 1 to find λ and $T_{\rm s}$ that maximize the throughput, while causing low interference. If the outputs λ^* and $T_{\rm s}^*$ from Algorithm 1 are such that the corresponding $\eta_{\rm c} \leq \hat{\eta_{\rm c}}$, then we use them as the threshold and packet time respectively. If not, starting with λ^* and $T_{\rm s}^*$, we use a modified Algorithm 1 where in the inner loop for optimizing $T_{\rm s}$, each time the $T_{\rm s}$ obtained at the end of the loop is such that corresponding $\eta_{\rm c} > \hat{\eta_{\rm c}}$, we lower $T_{\rm s}$ to the solution of $\hat{\eta_{\rm c}} = \frac{1}{L} \cdot \frac{{\rm E}[T_{\rm c}]}{{\rm E}[T_N]}$, obtained by Newton's method. Since there is a risk that at the end of the outer loop, the new λ and $T_{\rm s}$ are such that the throughput $\phi(\lambda, T_{\rm s})$ is lower, in such an event, we terminate the procedure and output the λ and $T_{\rm s}$ obtained at the end of previous iteration. (We perform at least one iteration.) If not, we iterate till convergence.

Clearly, the procedure has similar convergence guarantees as Algorithm 1 when $\eta_{\rm c}(\lambda^*, T_{\rm s}^*) \leq \hat{\eta_{\rm c}}$. In other cases, the throughput is lower and not guaranteed to be the best possible under the given interference constraints. A similar problem setup with similar solution structure and conclusions can be found in [25].

VII. Extensions to More General Scenarios

A. General Channel Statistics

Throughout the paper, we consider a homogeneous channel and system model where all channels have the same statistics, i.e., the rate distribution of SU is $F_R(r)$ for all channels and all the PUs have the same exponential distribution parameters a and b for the idle and busy times. The assumption can be justified due to consideration of the common scenario that the channels belong to the same licensed network [4], typically consisting of equal quality channels with equal usage constraints on the PUs.

To make our results more useful, we consider an extension to a heterogeneous scenario where the rate distribution of SU is not necessarily the same on the L channels, and given by $F_1(r)$, $F_2(r), \ldots, F_L(r)$ respectively. In such a scenario, it is natural to consider strategies where instead of exploring channels uniformly at random, we explore channels of higher quality more frequently. Accordingly, we consider schemes that explore the channels with unequal probabilities $p_1, p_2, \ldots, p_L$, where $\sum_{l=1}^{L} p_l = 1$. Using similar techniques as Section III, it can be shown for each channel l, the optimal stopping rule for deciding whether to transmit on that channel after probing is a threshold policy. Let the thresholds corresponding to the different channels be $\Lambda = (\lambda_1, \lambda_2, \ldots, \lambda_L)$ respectively. By noting that the probability of transmitting on channel i is proportional to $p_i(1 - F_i(\lambda_i))$, the corresponding throughput can be seen to be

$$x(\Lambda) = \frac{\frac{\sum_{i=1}^{L} p_i \int_{\lambda_i}^{\infty} r dF_i(r)}{\sum_{i=1}^{L} p_i (1-F_i(\lambda_i))} \mathrm{E}[T']}{\frac{\tau'}{\sum_{i=1}^{L} p_i (1-F_i(\lambda_i))} + \mathrm{E}[T_{\mathrm{tr}}]} = \frac{\sum_{i=1}^{L} p_i \int_{\lambda_i}^{\infty} r dF_i(r) \mathrm{E}[T']}{\tau' + \mathrm{E}[T_{\mathrm{tr}}] \sum_{i=1}^{L} p_i (1 - F_i(\lambda_i))}, \tag{17}$$

where $\tau' = \frac{\tau_{\mathrm{s}} + Q_{\mathrm{I}}' \tau_{\mathrm{p}}}{(\frac{b}{a+b}) e^{-a(\tau_{\mathrm{s}} + \tau_{\mathrm{p}})} (1 - P_{\mathrm{fa}})}$, and $\mathrm{E}[T']$, $\mathrm{E}[T_{\mathrm{tr}}]$ and Q_{I}' are given by Proposition 4.1. From the expression above, and as pointed out in [4], it is easy to see that for a given Λ, selecting $p_l = 1$ for the channel l that has the highest average throughput, and $p_i = 0$ for $i \neq l$ maximizes the throughput. However, for such a choice of p_i's, Equation (17) for throughput no longer holds because we assume that a large number of channels are explored and hence the exploration outcomes are independent. As a counter example, consider a scenario consisting of a large number of channels L, where $L - 1$ of them have equal rate distributions and the last channel has a slightly better rate distribution. And let the expected busy time $\frac{1}{b}$ be really large. In such a scenario, there is clear benefit of exploring all the channels, as opposed to stuck with the single best channel for a long time when PU is busy. Accounting for the higher order terms in the calculation of channel access times due to SU exploring previous channels is difficult and is not considered in this paper. Instead, we motivate and propose the following strategy for the heterogeneous case at hand. We first consider strategies that explores each channel i with equal probability $p_i = \frac{1}{L}$. To find Λ that maximizes throughput $x(\Lambda)$, we equate the derivative of $x(\Lambda)$ with respect to each λ_i to zero, and deduce that all λ_i are equal. Letting the common threshold to be λ, Equation (17) for throughput simplifies to

$$x(\Lambda) = \frac{\frac{\int_{\lambda}^{\infty} r dF(r)}{(1-F(\lambda))}\mathrm{E}[T']}{\frac{\tau'}{1-F(\lambda)} + \mathrm{E}[T_{\mathrm{tr}}]} \tag{18}$$

where $F(r) \triangleq \frac{1}{L}\sum_{i=1}^{L} F_i(r)$. This is therefore equivalent to a scenario that all channels have the same rate distribution equal to the average of the rate distributions of the original channels. And the best threshold λ^* can be found by optimization techniques considered earlier. While this strategy ensures that we are able to achieve the optimal performance corresponding to the average rate distribution, we may be able to get a better throughput using the following tweak. Suppose for this common choice of thresholds λ^*, we want to minimize the channel access time or equivalently maximize the channel access probability, by suitably selecting the probabilities $p_1, \ldots, p_L$. Note that this is not the same as maximizing the throughput, which would again lead to the degenerate solution $p_l = 1$ for the best channel l. The channel access probability is proportional to $\sum_{i=1}^{L} p_i(1 - F_i(\lambda^*))$, which is maximized when $p_l = \frac{1-F_l(\lambda^*)}{\sum_{i=1}^{L}(1-F_i(\lambda_i))}$, by Cauchy-Schwarz inequality or otherwise. This choice of p_i's also favors better channels, albeit in a moderate way. To summarize, we propose using the best threshold λ^* and packet time T_{s}^* as obtained in Algorithm 1 corresponding to the average rate distribution $F(r)$ and use channel exploration probabilities $p_i \propto (1 - F_i(\lambda^*))$.

We do not consider the heterogeneous case of different idle and busy parameters a and b across the channels, which is much more difficult because of the presence of exponential terms in throughput and it also affects the transmission times $\mathrm{E}[T']$ and $\mathrm{E}[T_{\mathrm{tr}}]$. A simple strategy in a heterogeneous case is to take $\frac{1}{a}$ and $\frac{1}{b}$ as the average idle and busy times across the channels. Lastly, while we only consider the rate distributions under Rayleigh fading, most of the results in this paper, including Algorithm 1, apply to other continuous, well-behaved rate distributions.

B. Multiple Secondary Users

In this paper, we only consider a setup with only one SU. In general, there can be multiple SUs sharing the channels with the PUs. In such a scenario, we not only have to consider collisions between each SU and the PUs, but also collisions among SUs. Let the number of SUs be M. When $M \ll L$, one way to extend the scheme and results in our paper to this scenario is to divide the L channels into M disjoint groups, each having L/M channels. And each SU exclusively explores channels in one of these groups. In such a case, each SU can use the scheme presented in this paper and the throughput optimality results hold as is. However, if M is larger than L, such a grouping is not possible. Moreover, even if M is smaller, but comparable to L, the number of channels L/M is no longer large, which is used as an assumption for the calculation of channel access times and throughput. The throughput is lower since the channel access time is higher due to the SU exploring a previous channel where PU may still be busy or rate may still be low.

An alternative is to consider schemes where each SU explores all the L channels uniformly at random. Assuming L is large, we can now neglect the higher order terms in the channel access time due to re-exploration of channels, which happens very infrequently. If $M \ll L$, we may neglect collisions with other SUs as well. If M is comparable to L, while we can still neglect excess access time due to re-exploration, we do need to address the excess due to collisions with other SUs. Accounting for excess times is beyond the scope of this paper and will be considered in a future work [26] along with other concerns like cooperation and fairness across the SUs.

Note that if M is even moderately larger than L, we need to consider other standard techniques for multiple access and explore channels infrequently. For example, the m-th SU may explore channels with probability $p_m < 1$ and remain inactive at other times. If p_m is small enough that collisions among SUs are infrequent, we can again directly use the results in this paper with the channel access times now increased by a factor of $\frac{1}{p_m}$.

VIII. Numerical Results

We present numerical results to evaluate the performance of our proposed scheme. Unless otherwise stated, the values of the various parameters used are $\rho = 10$, $\tau_{\rm s} = 20$ ms, $\tau_{\rm p} = 30$ ms, $\frac{1}{a} = 500$ ms, $\frac{1}{b} = 666.67$ ms, $P_{\rm fa} = 0.1$ and $P_{\rm md} = 0.05$. For simplicity, we assume that the optimal transmission time meets the interference requirements, i.e., the $T_{\rm s}^*$ obtained in Algorithm 1 satisfies $\eta_{\rm c} \leq \hat{\eta_{\rm c}}$. If not, the easy modification mentioned in Section VI can be used.

We study the performance of the proposed channel access scheme as a function of the key operational parameters. Specifically, we examine the maximal throughput x^*, the optimal threshold λ^* and the optimal transmission time $T_{\rm s}^*$ as a function of $\frac{1}{a}$, $P_{\rm fa}$ and $P_{\rm md}$ when each one of these is varied while keeping others fixed.

The dynamic behavior of PU directly affects SU's performance. The effect of PU's average idle time $\frac{1}{a}$ on SU's throughput is shown in Fig. 3. When $\frac{1}{a}$ increases, since SU has the opportunity to transmit for longer times, we observe that the optimal threshold λ^* and transmission time T_{s}^* increase, and consequently the throughput x^* increases.

Sensing errors have a negative impact on the performance of the proposed scheme. The impact of sensing errors in the form of various values of false alarm probabilities P_{fa} is shown in Fig. 4 and for misdetection probabilities P_{md} is shown in Fig. 5. Note that P_{fa} and P_{md} are decreasing functions of τ_{s}. But they are also functions of other channel and system parameters, *e.g.,* the channel bandwidth, the SNR of PU at SU's receiver, and the detection threshold used in sensing systems based on energy detection [4], [27]. Even though the value of τ_{s} is fixed in our numerical results, we attribute the different values of P_{fa} and P_{md} implicitly to the remaining parameters.

In Fig. 4, when P_{fa} increases, x^* decreases as expected whereas T_{s}^* increases. The reason for the increment in T_{s}^* is that when P_{fa} is high, i.e., when PU is detected as idle less often, SU increases its transmission time whenever it gets the chance to transmit. Similarly, λ^* decreases when P_{fa} increases because the transmission opportunity is smaller and thus λ^* is small so that transmission can take place more readily.

In Fig. 5, when P_{md} increases, x^* decreases similar to the effect of P_{fa}. Unlike the case of P_{fa}, when P_{md} increases, T_{s}^* decreases to reduce the amount of collision and data loss when PU returns but is misdetected as idle. As in the case of P_{fa}, for small values of P_{md}, when P_{md} increases, λ^* decreases to facilitate channel access more readily. However, for high values of P_{md}, there may be an increase in λ^* so that transmission is carried out at a higher rate, albeit less often, thereby also reducing frequent collision and data loss due to misdetection. This phenomenon is not observed in Fig. 5.

It is of interest to compare our proposed scheme for opportunistic channel-aware spectrum access with periodic sensing to other schemes. One such comparison is shown in Fig. 6, our scheme is compared with the one without periodic sensing and one without probing. To obtain a fair comparison, the transmission time T_{s}' used for the scheme without periodic sensing is the same as expected time of transmission $\mathrm{E}[T_{\mathrm{tr}}]$ for the periodic sensing scheme. A threshold based channel access strategy is used for the scheme without periodic sensing, where the threshold that maximizes throughput is derived using optimal stopping theory, similar to the scheme with periodic sensing. The results show that significant throughput gains are achieved for the proposed scheme over the schemes without periodic sensing and without probing.

Next, we evaluate the benefit of optimizing the transmission time $T_{\rm s}$ – if it is too large, the return of PU will lead to loss of the entire packet, and if it too small, we spend too much time in periodic sensing. In Fig. 7, we compare our proposed scheme with a scheme without optimal transmission time, arbitrarily set to a large value of 500 ms and small value of 50 ms. We observe significant improvements by optimizing $T_{\rm s}$ as shown in the figure.

To further demonstrate the benefit of our proposed scheme, we compare our scheme with a POMDP-based scheme in [3]. For fair comparison, we set the rates of all channels to $\mathrm{E}[R]$, slot length to $\tau_{\rm s}+T_{\rm s}$ and the transition probabilities $p_{\text{busy}\rightarrow\text{idle}} = b(\tau_{\rm s}+T_{\rm s})$ and $p_{\text{idle}\rightarrow\text{busy}} = a(\tau_{\rm s}+T_{\rm s})$ so that average idle and busy times of PUs are $1/a$ and $1/b$. POMDP-based schemes are popular in the literature for solving the opportunistic spectrum access problem. Such schemes dynamically track the idle state of various channels in a slotted system and maximize throughput by exploiting the spectrum opportunities. Our scheme fully utilizes the idle state of the channels it accesses by periodic sensing, and together with exploitation of channel quality information and optimization of transmission time, it outperforms the POMDP-based scheme as seen in Fig. 8.

IX. CONCLUSIONS

In this paper, we proposed an opportunistic channel access framework where the transmissions are interleaved with periodic sensing. For the proposed scheme, we obtained the optimal threshold and the optimal transmission period that jointly maximize the average throughput. We consider the effect of sensing errors throughout the analysis. Numerical results show that our scheme can offer a much higher throughput than other well-known schemes. We also studied numerically the effect of some of the important channel and system parameters on our scheme as they vary over a range of values.

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APPENDIX A
PROOF OF PROPOSITION 3.1

The proof of Proposition 3.1 uses methods from optimal stopping theory [23] and closely follows a similar result in [8]. In order to maximize the average throughput $\frac{\mathrm{E}[R_N T']}{\mathrm{E}[T_N]} = \frac{\mathrm{E}[R_N]\mathrm{E}[T']}{\mathrm{E}[T'_N]+\mathrm{E}[T_{\mathrm{tr}}]}$, a standard technique [23, Ch. 6] is to consider for all $x \in (0,\infty)$, the reward function $Z_n(x) \triangleq R_n \mathrm{E}[T'] - x(T'_n + \mathrm{E}[T_{\mathrm{tr}}])$ and an optimal stopping rule $N(x)$ that maximizes the expected reward $\mathrm{E}[R_N T' - xT_N]$. Let the corresponding maximum reward be

$$V(x) \triangleq \sup_{N\in Q} E[Z_N(x)] = \sup_{N\in Q} \mathrm{E}[R_N T' - xT_N] = \mathrm{E}[R_{N(x)}T' - xT_{N(x)}].$$

The motivation behind considering the reward function $Z_n(x)$ is [23, Ch. 6, Th. 1], which states that if the maximum rate, i.e., throughput is $x^* \triangleq \sup_{N\in Q} \frac{\mathrm{E}[R_N T']}{\mathrm{E}[T_N]}$, then $V(x^*) = 0$, and furthermore, $N(x^*)$ is the stopping rule that maximizes throughput.

Using [23, Ch. 3, Th. 1], the existence of $N(x)$ is guaranteed if

$$\mathrm{E}[\sup_n Z_n(x)] < \infty \quad \text{and} \quad \limsup_{n\to\infty} Z_n(x) = -\infty \text{ a.s.}$$

We show that both these conditions are satisfied in our setup. We express the time spent in a round for successfully accessing a channel as $T'_n = \sum_{i=1}^{n}(K_i\tau_{\rm s}+K'_i\tau_{\rm p})$, where K_i and K'_i are the number of channels sensed and probed respectively to find the i-th idle channel, for $i = 1, 2, \ldots, n$. Note that the $K_1, K_2, \ldots, K_n$ are *i.i.d.* and have the same distribution as $K_{\rm s}$, and likewise $K'_1, K'_2, \ldots, K'_n$ are *i.i.d.* copies of $K_{\rm p}$. It is hence easy to see that $\limsup_{n\to\infty} \mathrm{E}[Z_n(x)] = -\infty$ almost surely. This is because both R_n, the channel rate under Rayleigh fading, and T', which is related to a geometrically distributed r.v., have finite mean and variance. Furthermore, $K_{\rm s} \geq 1$, $K_{\rm p} \geq 1$ and $x > 0$. We show that $\mathrm{E}[\sup_n Z_n(x)] < \infty$ by a similar reasoning. Observe that

$$\begin{aligned}\mathrm{E}[\sup_n Z_n(x)] \leq \mathrm{E}\big[\sup_n R_n T' - nx\epsilon(\mathrm{E}[K_{\rm s}]\tau_{\rm s} + \mathrm{E}[K_{\rm p}]\tau_{\rm p})\big] \\ + \mathrm{E}\big[\sup_n x\sum_{i=1}^{n}\epsilon(\mathrm{E}[K_{\rm s}]\tau_{\rm s} + \mathrm{E}[K_{\rm p}]\tau_{\rm p}) - (K_i\tau_{\rm s} + K'_i\tau_{\rm p})\big],\end{aligned}$$

for any $\epsilon \in (0, 1)$. The contribution due to $T_{\rm tr}$ is negative and safely ignored. Again using the fact R_n and T' are positive random variables with finite mean and variance, we use [23, Ch. 4, Th. 1 and Th. 2] to conclude that both the terms on the right hand side of above inequality are finite. Thus, the existence of $N(x)$ is guaranteed for all $x \in (0, \infty)$.

We proceed to find $N(x)$ and x^*. Using the *principle of optimality* [23, Ch. 3, Th. 3], an optimal stopping rule is

$$\begin{aligned}N(x) &= \min\{n \geq 1 : R_n\mathrm{E}[T'] - x(\mathrm{E}[T_{\rm tr}] + T'_n) \geq V(x) - xT'_n\} \\ &= \min\{n \geq 1 : R_n\mathrm{E}[T'] \geq V(x) + x\mathrm{E}[T_{\rm tr}]\},\end{aligned}$$

and the *optimality equation* [23, Ch. 3, Th. 2] gives

$$V(x) = \mathrm{E}\big[\max\big\{R_1\mathrm{E}[T'] - x\mathrm{E}[T_{\rm tr}], V(x)\big\} - x(K_1\tau_{\rm s} + K'_1\tau_{\rm p})\big].$$

Using $V(x^*) = 0$ and the above expressions for $N(x)$ and $V(x)$, we conclude that the stopping rule that maximizes the throughput is

$$N(x^*) = \min\big\{n \geq 1 : R_n \geq x^*\frac{\mathrm{E}[T_{\rm tr}]}{\mathrm{E}[T']}\big\},$$

and the maximal throughput x^* is a solution for x in

$$\mathrm{E}\Big[\big(R_n - x\frac{\mathrm{E}[T_{\rm tr}]}{\mathrm{E}[T']}\big)^+\Big] = \frac{x(\mathrm{E}[K_{\rm s}]\tau_{\rm s} + \mathrm{E}[K_{\rm p}]\tau_{\rm p})}{\mathrm{E}[T']}.$$

Lastly, we show that the above equation for x^* has a unique solution. We perform a change of variable $\lambda \triangleq x\frac{\mathrm{E}[T_{\mathrm{tr}}]}{\mathrm{E}[T']}$ and equivalently show that there is a unique solution for λ in

$$\mathrm{E}\big[(R_n - \lambda)^+\big] = \lambda\frac{(\mathrm{E}[K_{\mathrm{s}}]\tau_{\mathrm{s}} + \mathrm{E}[K_{\mathrm{p}}]\tau_{\mathrm{p}})}{\mathrm{E}[T_{\mathrm{tr}}]}. \tag{19}$$

The left hand side of equation (19) can be written as

$$g(\lambda) \triangleq \mathrm{E}\big[(R_n - \lambda)^+\big] = \int_{\lambda}^{\infty}(r-\lambda)f_R(r)dr.$$

Clearly, $g(\lambda)$ is continuous and decreases from $\mathrm{E}[R_n]$ to 0, since $f_R(r)$ is positive, continuous and differentiable and hence for $\lambda_1 < \lambda_2$, we have

$$\begin{aligned} g(\lambda_2) - g(\lambda_1) &= \int_{\lambda_2}^{\infty}(r-\lambda_2)f_R(r)dr - \int_{\lambda_1}^{\infty}(r-\lambda_1)f_R(r)dr \\ &= \int_{\lambda_2}^{\infty}(\lambda_1-\lambda_2)f_R(r)dr - \int_{\lambda_1}^{\lambda_2}(r-\lambda_1)f_R(r)dr \\ &\le 0. \end{aligned}$$

The right hand side of equation (19), $\lambda\frac{(\mathrm{E}[K_{\mathrm{s}}]\tau_{\mathrm{s}} + \mathrm{E}[K_{\mathrm{p}}]\tau_{\mathrm{p}})}{\mathrm{E}[T_{\mathrm{tr}}]}$ is continuous and increasing from 0 to ∞. Hence, the equation (19) has a unique solution in λ. Note that the solution $\lambda = \lambda^*$ is the threshold in the optimal stopping rule, i.e., the throughput maximizing stopping rule is $\{n \ge 1 : R_n \ge \lambda^*\}$ and the maximum throughput is $x^* = \lambda^*\frac{\mathrm{E}[T']}{\mathrm{E}[T_{\mathrm{tr}}]}$.

APPENDIX B
PROOF OF PROPOSITION 4.1

The proof uses properties of exponential and geometric distributions, especially the memoryless property of exponential distributions.

Expected Effective Transmission Time ($\mathrm{E}[T']$)

If K is the number of packets transmitted successfully by SU in a round, then

$$\begin{aligned} \Pr(K=k) &= e^{-a(kT_{\mathrm{s}}+(k-1)\tau_{\mathrm{s}})}\cdot(1-P_{\mathrm{fa}})^{k-1}\cdot\Big(\big(1-e^{-a(T_{\mathrm{s}}+\tau_{\mathrm{s}})}\big)+e^{-a(T_{\mathrm{s}}+\tau_{\mathrm{s}})}P_{\mathrm{fa}}\Big) \\ &= e^{-aT_{\mathrm{s}}}\Big(e^{-a(T_{\mathrm{s}}+\tau_{\mathrm{s}})}(1-P_{\mathrm{fa}})\Big)^{k-1}\cdot\Big(1-e^{-a(T_{\mathrm{s}}+\tau_{\mathrm{s}})}(1-P_{\mathrm{fa}})\Big). \end{aligned}$$

This is because PU should be idle during the transmission of first k packets, i.e., for a time of $kT_{\mathrm{s}} + (k-1)\tau_{\mathrm{s}}$ from the start of transmission. Also, there should be no false alarm in the first $k-1$ sensing phases. Lastly, either PU should return during the following sensing phase or packet transmission (i.e., the following duration of $\tau_{\rm s}+T_{\rm s}$), or the transmission should be terminated due to false alarm if PU does not return. The above distribution of K closely resembles a geometric distribution with parameter $e^{-a(\tau_{\rm s}+T_{\rm s})}(1-P_{\rm fa})$ and $\mathrm{E}[K]=\sum_k k\Pr(K=k)=\frac{e^{-aT_{\rm s}}}{1-e^{-a(T_{\rm s}+\tau_{\rm s})}(1-P_{\rm fa})}$. Since the packet duration is $T_{\rm s}$, we have $\mathrm{E}[T']=\frac{T_{\rm s}\cdot e^{-aT_{\rm s}}}{1-e^{-a(T_{\rm s}+\tau_{\rm s})}(1-P_{\rm fa})}$.

Derivation of Expected Time for successfully accessing the channel ($\mathrm{E}[T'_N]$)

The number of different channels K that are explored, i.e., sensed and possibly probed, for finding good channel in a round is distributed geometrically as $\Pr(K=k)=(1-z)^{k-1}z$ for $k\in\{1,2,\ldots\}$, where $z=\frac{b}{a+b}e^{-a(\tau_{\rm s}+\tau_{\rm p})}(1-P_{\rm fa})(1-F_R(\lambda))$. This is because a good channel must satisfy the following conditions:

1) PU should be idle at the start of the sensing phase, the probability of which is $\frac{b}{a+b}$.
2) PU should continue to be idle during the duration $\tau_{\rm s}+\tau_{\rm p}$ of sensing and probing, which happens with probability $e^{-a(\tau_{\rm s}+\tau_{\rm p})}$. (This is the conditional probability, given that the channel was idle to begin with.)
3) There should be no false alarm, which happens with probability $(1-P_{\rm fa})$.
4) The rate of the channel should be higher than threshold and $\Pr(R>\lambda)=1-F_R(\lambda)$.

If any of these conditions are not satisfied, SU proceeds to explore another channel. Hence,

$$\mathrm{E}[K]=\frac{1}{z}=\frac{1}{\frac{b}{a+b}e^{-a(\tau_{\rm s}+\tau_{\rm p})}(1-P_{\rm fa})(1-F_R(\lambda))}. \tag{20}$$

Of these K channels, the first $K-1$ are bad. Probability that a channel is bad and probed, is

$$p_{\rm bad,probe}=\Big(\frac{a}{a+b}\Big)P_{\rm md}+\Big(\frac{b}{a+b}\Big)\Big(e^{-a\tau_{\rm s}}(1-P_{\rm fa})\big((1-e^{-a\tau_{\rm p}})+e^{-a\tau_{\rm p}}F_R(\lambda)\big)+(1-e^{-a\tau_{\rm s}})P_{\rm md}\Big).$$

Here, the first summand is the probability of the case that PU is busy to begin with, but is misdetected as idle. The second summand corresponds to the case when PU is idle to begin with. There are two subcases here when the channel is probed. In the first subcase, PU is idle during sensing duration of $\tau_{\rm s}$ and there is no false alarm. But the channel is bad either because PU returns during probing duration of $\tau_{\rm p}$ or the rate is low. The second subcase is that PU returns during the sensing phase but is misdetected as idle. All the K explored channels are sensed for a duration of $\tau_{\rm s}$. The $K-1$ bad channels are probed with probability $\frac{p_{\rm bad,probe}}{1-z}$ (which is the conditional probability that a channel is probed given that it is bad). If K' of the $K-1$ bad channels are probed, then $\mathrm{E}[K']=\mathrm{E}[K-1]\frac{p_{\rm bad,probe}}{1-z}$. The K-th channel which is good, is also probed for a duration of τ_p. By putting together these observations, we have

$$\begin{aligned}
\mathrm{E}[T'_N] &= \mathrm{E}[K\tau_\mathrm{s} + K'\tau_\mathrm{p} + \tau_\mathrm{p}] = \mathrm{E}[K]\tau_\mathrm{s} + \big(\mathrm{E}[K-1]\frac{p_\mathrm{bad,probe}}{1-z} + 1\big)\tau_\mathrm{p} \\
&= \frac{1}{z}\tau_\mathrm{s} + \Big((\frac{1}{z}-1)\frac{p_\mathrm{bad,probe}}{1-z} + 1\Big)\tau_\mathrm{p} = \frac{\tau_\mathrm{s} + (p_\mathrm{bad,probe} + z)\tau_\mathrm{p}}{z} \\
&= \frac{\tau_\mathrm{s} + \big((\frac{a}{a+b})P_\mathrm{md} + (\frac{b}{a+b})\big((1-e^{-a\tau_\mathrm{s}})P_\mathrm{md} + e^{-a\tau_\mathrm{s}}(1-P_\mathrm{fa})\big)\big)\tau_\mathrm{p}}{(\frac{b}{a+b})e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1-P_\mathrm{fa})(1-F_R(\lambda))} \\
&= \frac{\tau_\mathrm{s} + Q'_\mathrm{I}\tau_\mathrm{p}}{(\frac{b}{a+b})e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1-P_\mathrm{fa})(1-F_R(\lambda))}.
\end{aligned}$$

The number of channels explored, K_s, for finding a channel that can be probed completely is distributed geometrically with parameter $\frac{b}{a+b}e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1-P_\mathrm{fa})$ and hence

$$\mathrm{E}[K_\mathrm{s}] = \frac{1}{\frac{b}{a+b}e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1-P_\mathrm{fa})}. \tag{21}$$

If K_p out of these K_s channels are considered for probing, then

$$\mathrm{E}[K_\mathrm{p}] = \frac{\big(\frac{a}{a+b}\big)P_\mathrm{md} + \big(\frac{b}{a+b}\big)\big((1-e^{-a\tau_\mathrm{s}})P_\mathrm{md} + e^{-a\tau_\mathrm{s}}(1-P_\mathrm{fa})\big)}{(\frac{b}{a+b})e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1-P_\mathrm{fa})} = \frac{Q'_\mathrm{I}}{(\frac{b}{a+b})e^{-a(\tau_\mathrm{s}+\tau_\mathrm{p})}(1-P_\mathrm{fa})},$$

using arguments similar to that for calculating $\mathrm{E}[T'_N]$ earlier.

Expected Time for Transmission with Periodic Sensing ($\mathrm{E}[T_\mathrm{tr}]$)

The time T_tr is spent by SU in each round for transmitting its packets along with periodic sensing, until it detects the return of PU, either correctly or due to false alarm. Observe that T_tr is a multiple of $(T_\mathrm{s}+\tau_\mathrm{s})$ since SU alternately transmits a packet followed by sensing for PU's return. In the case when there is no misdetection, i.e., $P_\mathrm{md} = 0$, the probability that the transmission lasts for $k \geq 1$ periods is $(1-z)^{k-1}z$ where $z = (1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}) + e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}P_\mathrm{fa} = 1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1-P_\mathrm{fa})$. Hence, the expected number of such periods is $\frac{1}{z} = \frac{1}{1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1-P_\mathrm{fa})}$. When $P_\mathrm{md} > 0$, the number of additional periods for which the transmission carries is distributed geometrically with parameter $1-P_\mathrm{md}$ and the expected number of such periods is $\frac{P_\mathrm{md}}{1-P_\mathrm{md}}$. However, additional periods due to misdetection can occur only when PU returns, and not in the case when PU does not return and transmission terminates due to false alarm. The fraction of times transmission terminates due to PU returning is $\frac{1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}}{(1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})})+e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}P_\mathrm{fa}} = \frac{1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}}{1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1-P_\mathrm{fa})}$. Thus,

$$\begin{aligned}
\mathrm{E}[T_\mathrm{tr}] &= \frac{T_\mathrm{s}+\tau_\mathrm{s}}{1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1-P_\mathrm{fa})} + \frac{(1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})})}{1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1-P_\mathrm{fa})}\frac{P_\mathrm{md}}{1-P_\mathrm{md}}(T_\mathrm{s}+\tau_\mathrm{s}) \\
&= \frac{1-P_\mathrm{md}e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}}{1-P_\mathrm{md}}\frac{(T_\mathrm{s}+\tau_\mathrm{s})}{1-e^{-a(T_\mathrm{s}+\tau_\mathrm{s})}(1-P_\mathrm{fa})}.
\end{aligned}$$

Expected Transmission Rate ($\mathrm{E}[R_N]$)

Under a stopping rule $N = \min\{n : R_n \geq \lambda\}$ that is a pure threshold policy,

$$\mathrm{E}[R_N] \;=\; \mathrm{E}[R|R > \lambda] = \frac{\int_{\lambda}^{\infty} r dF_R(r)}{1 - F_R(\lambda)}. \tag{22}$$

Appendix C

Proof of Proposition 4.2

The proof is along the same lines as that of [8, Prop. 3.4]. Using (5), (8), (9), and Proposition 3.1, it follows that

$$\lambda^* = \max_{\lambda} \psi(\lambda) \;=\; \psi(\lambda^*). \tag{23}$$

Proposition 3.1 and (23) imply that the functions $y = \lambda$ and $y = \psi(\lambda)$ for $\lambda > 0$ intersect only at $\lambda = \lambda^*$. Together with $\psi(0) > 0$, we have

$$\psi(\lambda) > \lambda \text{ for } \lambda < \lambda^*, \quad \text{and} \quad \psi(\lambda) < \lambda \text{ for } \lambda > \lambda^*. \tag{24}$$

If $\lambda_0 > \lambda^*$, then $\lambda_1 = \psi(\lambda_0) \leq \psi(\lambda^*) = \lambda^*$, i.e., equivalent to starting with $\lambda_1 \leq \lambda^*$. Hence, we assume that $\lambda_0 \leq \lambda^*$. Then, by induction for $k \in \{0, 1, 2, \ldots\}$, we have $\lambda_{k+1} = \psi(\lambda_k) \geq \lambda_k$. Furthermore, $\lambda_{k+1} = \psi(\lambda_k) \leq \psi(\lambda^*) = \lambda^*$ for all k. Thus, $\{\lambda_k\}_{k=0}^{\infty}$ is a monotonically increasing sequence upper bounded by λ^*, and therefore converges to a limit, say λ_∞.

We finally show that $\lambda_\infty = \lambda^*$. We have $\psi(\lambda_k) - \lambda_k = \lambda_{k+1} - \lambda_k$. By taking the limit $k \to \infty$ on both sides, we have $\psi(\lambda_\infty) - \lambda_\infty = 0$. Since $\psi(\lambda) - \lambda = 0$ has a unique solution λ^*, we conclude $\lambda_\infty = \lambda^*$.

Appendix D

Proof of Proposition 5.1

From (13), we observe that for any given λ, $\phi'_{T_\mathrm{s}}(\lambda, T_\mathrm{s}) \triangleq \frac{\partial}{\partial T_\mathrm{s}} \phi(\lambda, T_\mathrm{s})$ is continuous in T_s. We have $\phi'_{T_\mathrm{s}}(\lambda, T_\mathrm{s})|_{T_\mathrm{s}=0} = \frac{c_6 - c_8}{(c_6 - c_8)^2} > 0$ since $c_6 = c_2 + c_4\tau_\mathrm{s} > c_2 c_3 + c_4 c_5 \tau_\mathrm{s} = c_8$. It is easy to see that $\phi'_{T_\mathrm{s}}(\lambda, T_\mathrm{s})|_{T_\mathrm{s}=\infty} = -\infty$. If $\phi'_{T_\mathrm{s}}(\lambda, T_\mathrm{s}) = 0$ for some value of T_s, then it satisfies (14). We show that (14) has a unique solution for $T_\mathrm{s} > 0$, *i.e.,* the function $\zeta(T_\mathrm{s}) \triangleq c_6 - c_8 e^{-aT_\mathrm{s}} - ac_6 T_\mathrm{s} - ac_4 T_\mathrm{s}^2$ has only one positive root. To see this, we observe that $\zeta(T_\mathrm{s})$ is concave since $\frac{\partial^2}{\partial T_\mathrm{s}^2} \zeta(T_\mathrm{s}) = -a^2 c_8 e^{-aT_\mathrm{s}} - 2ac_4 < 0$. Hence, it can have at most two roots. Furthermore, $\zeta(0) = c_6 - c_8 > 0$. Hence, it has exactly one positive and negative root. Let this positive root be T_s^*. Combining these arguments, it follows that for $0 < T_\mathrm{s} < T_\mathrm{s}^*$, $\phi'_{T_\mathrm{s}}(\lambda, T_\mathrm{s}) > 0$, i.e., $\phi(\lambda, T_\mathrm{s})$ is increasing. And for $T_\mathrm{s} > T_\mathrm{s}^*$, $\phi(\lambda, T_\mathrm{s})$ is decreasing. Thus, $\phi(\lambda, T_\mathrm{s})$ is maximized at $T_\mathrm{s} = T_\mathrm{s}^*$.

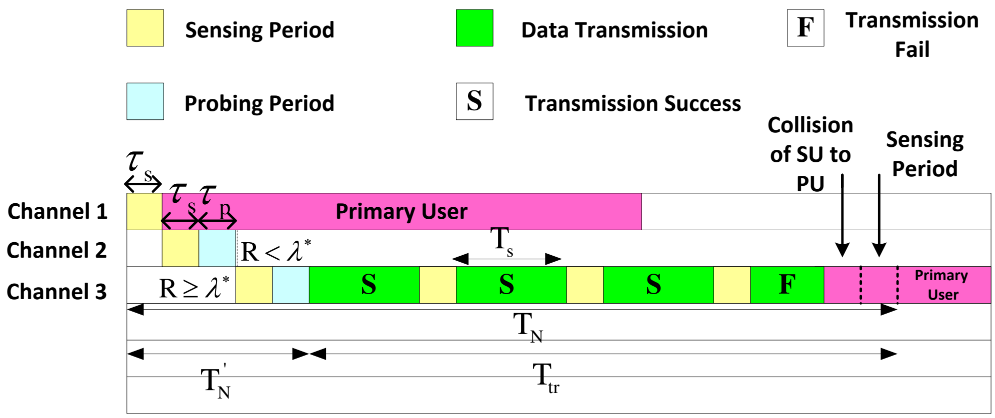


Fig. 1. A sample realization of channel sensing, probing and data transmission with PU returns

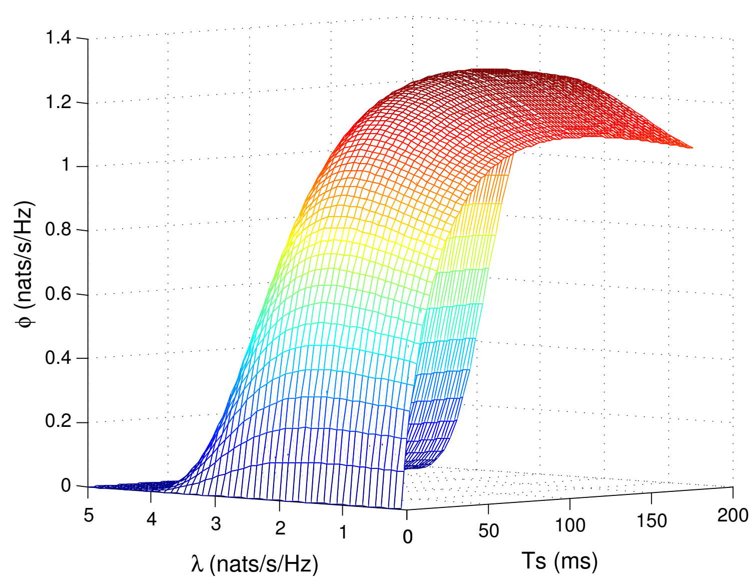
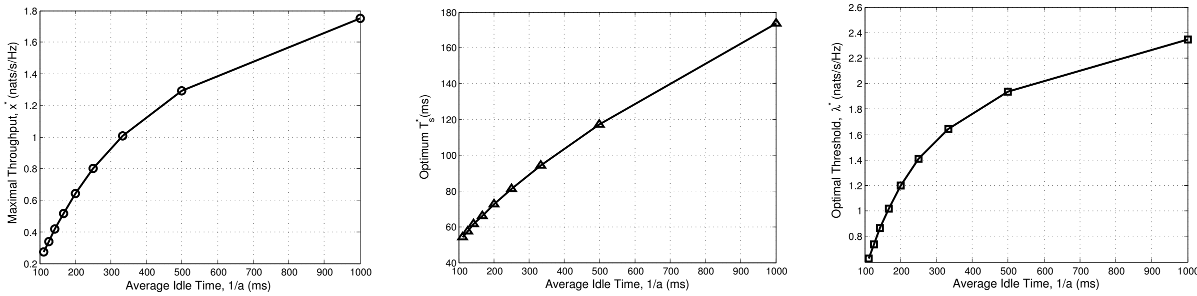
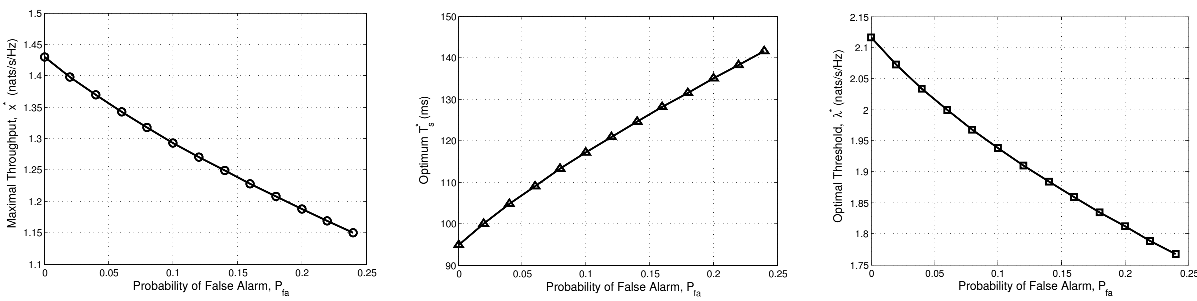


Fig. 2. The average throughput $\phi(\lambda, T_{\rm s})$ as a function of threshold λ and transmission time $T_{\rm s}$



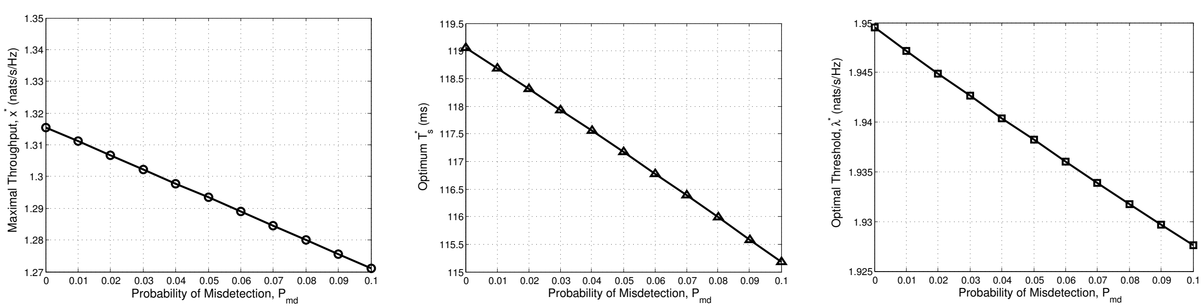
(a) (b) (c)

Fig. 3. The effect of average idle time: (a) Maximal throughput x^* versus average idle time, $\frac{1}{a}$ (b) Optimal $T_{\rm s}^*$ versus average idle time, $\frac{1}{a}$ (c) Optimal threshold λ^* versus average idle time, $1/a$



(a) (b) (c)

Fig. 4. The effect of false alarm ($P_{\rm fa}$ varies but $P_{\rm md} = 0.05$): (a) Maximal throughput x^* versus probability of false alarm, $P_{\rm fa}$ (b) Optimal $T_{\rm s}^*$ versus probability of false alarm, $P_{\rm fa}$ (c) Optimal threshold λ^* versus probability of false alarm, $P_{\rm fa}$



(a) (b) (c)

Fig. 5. The effect of misdetection ($P_{\rm md}$ varies but $P_{\rm fa} = 0.1$): (a) Maximal throughput x^* versus probability of misdetection, $P_{\rm md}$ (b) Optimal $T_{\rm s}^*$ versus probability of misdetection, $P_{\rm md}$ (c) Optimal threshold λ^* versus probability of misdetection, $P_{\rm md}$

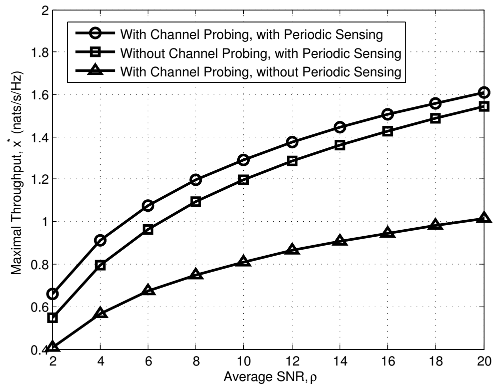


Fig. 6. Comparison between our scheme and one without channel probing and one without periodic sensing: Maximal throughput x^* versus average SNR ρ

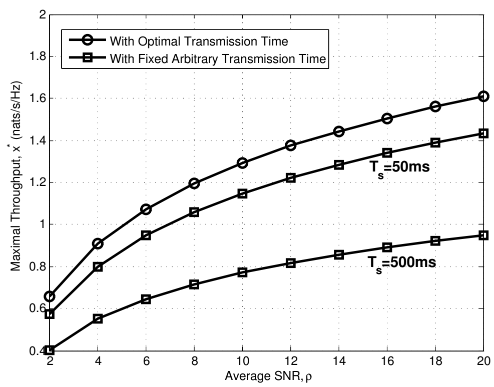


Fig. 7. Comparison between scheme with and without optimal transmission time: Maximal throughput x^* versus average SNR ρ. Note that optimal transmission time T_s^* varies with ρ.

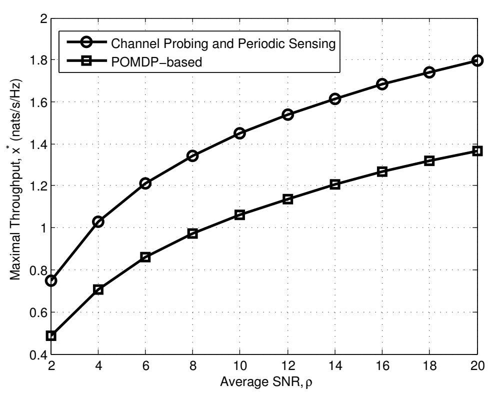


Fig. 8. Comparison between our proposed channel-aware with periodic sensing scheme and POMDP-based scheme : Maximal throughput x^* versus average SNR ρ